On Rational Decision with Subjective Probability in Design

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Abstract

Rationality has been regarded by many in the school of normative decision making as the sole criterion for good decisions. Its foundation is the subjective and personal interpretation of probability. In this brief, I argue that rational decision with personal probability is based on several assumptions and the validity of these assumptions is vastly debatable. First, I show that the history of the probability theory is the debate between interpretations of probability. The conclusion that subjective probability is the only interpretation will simply exclude the theory itself from applications in physical sciences such as engineering. Then, it is demonstrated that the assumptions of the Dutch book arguments, which are used as the philosophical base for rational or coherent decisions, are subject to debate. At the end, it is argued that the rational decision making based on the expected utility theory is purely on the coherent preference and nothing else. The further restriction on probability measures is a misinterpretation of the theory of preference. The view that rational design is equivalent to coherent design decisions is an over-simplification on real-world engineering issues.

1 Introduction

Making decisions under uncertainty is regarded as an important part of engineering design process. How to make good design decisions to achieve rational design has been a widely discussed topic in engineering design community [10]. One school of researchers strongly request that the expected utility theory based on the subjective or personal probability should be the sole criterion and approach to make design decisions.

In this brief, I will argue that objective and subjective interpretations have co-existed since the start of the probability theory. The grand debate between the two schools is not a recent activity. Blindly rejecting the existence of objective probability or aleatory uncertainty simply makes probability theory worth much less in practical applications in physical sciences, including engineering. Furthermore, axiomatic probability theory has been argued as the behavior of rational agent in the normative context. Particularly the Dutch book arguments have been used as the philosophical base of rationality. Yet, assumptions of the Dutch book arguments need also be aware of. The rationality basis in the arguments is questionable. Additionally, it should also be aware of that the expected utility theory is a *theory of preference*. The misconception of its additional restriction on probability measures is a narrow interpretation of the theory. The original theory and its extension to the multi-attribute utility theory enable us to derive personal probabilities from the specified preferences. It will be demonstrated that rational decisions can be made without specifying precise values of personal probabilities. Rationality in decision making is more of preferences than of probabilities. Lastly, it is argued that rational design and rational decision making are not equivalent. There is a need to define rational design more formally.

2 Interpretations of Probability Theory

The calculus of probability is dated back to July 1654 when Blaise Pascal wrote Pierre de Fermat on the dice game problem with the discussion of the outcome prediction [6]. At its infancy stage in the next 100 years, the meaningful applications of probability were in the areas of annuities and insurance where the calculation of mortality based on tabulated data was of interest, as well as astronomy where changeable atmospheric conditions along with inaccurate instruments and imperfect eyesight required multiple measurements of star and planet positions. Frequency was the only interpretation when the probability theory was started.

Subjective interpretation of probability was initiated by Thomas Bayes in his study of inverse probability [2]. It was largely unknown until Marquis de Laplace popularized the concept of degrees of belief in the early nineteenth century [9]. Laplacean probability intended to conflate the epistemic and frequency interpretations. Its concepts of Gaussian distribution and least squares were successfully applied in sciences and well accepted by physicists. Yet it was criticized soon by frequentists, particularly on its *principle of insufficient reason* as "equal distribution of ignorance". Since then, the philosophical debate between the two major schools of interpretations never ends.

Under the influence of the Laplacean probability, the physicists in the nineteenth century largely believed that the world is strictly deterministic. Probability is just a mental state as the reflection of our incomplete knowledge. For instance, Poincaré believed physical causes determined all events and there was nothing inherently random about the universe. Boltzmann used probability to describe a large number of molecules without uncertainty. The landscape quickly changed in the early twentieth century when quantum mechanics emerged as the fundamental description of the physical world. The irreducible dispersion described by the wave functions as probability distributions in the Schrödinger equation is able to explain various quantum phenomena such as the discrete energy state, photoelectric effect, quantum superposition and entanglement, etc. It is widely accepted nowadays that objective probability or inherent randomness does exist. The "hidden parameters" explanation as a deterministic view of the world is incompatible with fundamental postulates of quantum mechanics. The typical view of modern physicists is what von Neumann wrote [17] (p.323): there are no ensembles which are free from dispersion.

3 Assumptions of The Dutch Book Arguments

The Dutch book argument of rationality was championed by Bruno de Finetti with his coherent prevision interpretation. The main theme is as follows. For any proposition A, there is a number $p(A)$ such that you are willing to accept any bet with betting quotient $p(A)$. That is, based on your degree of belief $p(A)$ that the proposition A is true, you are willing to buy the bet "A is true" at a cost of $p(A)$. $p(A)$ is looked as a fair price by you. You are also willing to sell the bet with the price $p(A)$ too. In order to ensure the rationality, i.e. there is no sure loss, the sufficient condition is that the quotient p should satisfy the three Kolmogorov axioms of probability. A set of bets in which the corresponding p's do not satisfy any of the axioms will always lead to a sure loss, which is called a *Dutch book*. The Dutch book arguments were also extended to other laws of probability such as conditional probability.

The first simple example of the Dutch book arguments with the normalization axiom violation is as follows. Let H denote that a coin will land its head on the next toss and H^c be its supplement.

Suppose that you post the quotients $p(H) = 0.6$ and $p(H^c) = 0.5$, which means that you are willing to pay \$0.6 for a bet on H that pays you \$1 if H occurs. Similarly, you are willing to pay \$0.5 for a bet against H that pays you \$1 if H does not occur. A bookie sells you one bet on H and another on H^c, who collects \$1.1 from you and immediately returns \$1 to you. No matter whether H or H^c occurs, you have a sure loss.

The second example is with the finite additivity axiom. Let N denote that the Netherlands wins the next world cup and S that Spain wins the next world cup. You post the betting quotients as $p(N) = 0.5$, $p(S) = 0.2$, and $p(N \vee S) = 0.6$. A bookie sells you a bet on N and another on S and also buys a bet from you on $N \vee S$ each for \$1. The bookie will always make a profit of \$0.1.

Beyond the above two simple examples, the Dutch book arguments have also been extended to conditionalization and utility. It should be noted that several assumptions have been made in the Dutch book arguments, listed as follows.

First, you must post all betting quotients of all events at the beginning. The complete knowledge of all outcomes, including their relationships of dependency and mutual exclusiveness, in the world of the discourse is fully expressed in your belief. The degrees of belief on all possible outcomes should be explicit without any hesitation and indeterminacy. Doubt and vagueness are not permitted regardless of your personality. Based on the arguments, even if you have a modest and diffident personality, you should choose the subjective probability that is not part of your true subjective feeling.

Second, you must accept all bets anyone wants to make at your posted quotients. With the listed prices, decisions of either 'buy' or 'sell' should be made immediately, since the probability denotes the fair price at which both you will buy the bet and you will sell the bet. There are only two possible options of decisions. There is no option of 'not buy' or 'not sell'. The arguments have the assumption that a decision of 'sell' is the same as 'not buy'. In our real world that promotes 'value-adding' activities, such non-value-adding activities are not deemed as rational. It is intriguing that in the arguments a rational agent will make no profit through his/her buy and sell activities, whereas there is always a cunning bookie who will always bankrupt you if your posted quotients do not satisfy the axioms of probability.

Third, the values of the multiple bets you place are independent so that the bet placed earlier does not affect the following ones. There is no interference between bets, and the values of rewards are linear. These assumptions are particularly strong for the additivity of belief [13]. In the additivity example above, a rational agent is supposed to post the quotients $p(N) = 0.5$, $p(S) = 0.2$, and $p(N \vee S) = 0.7$. However, whoever wins the next cup, the agent will not gain anything. It will be very difficult, if not impossible, to convince an agent that does not just enjoy the gambling process to buy such bets. The whole argument of rational behavior is only based on the *fair* price instead of a *favorable* one. The additional assumption appears to be that neither loss and nor gain is rational. In contrast, a sub-additivity belief where the price for the union of the mutually exclusive is sufficiently larger than the sum of the prices for individual bets appears to be more attractive. Buying $N \vee S$ in this case is dependent on the previously bought individual bets on N and S.

Fourth, it is assumed that Dutch book susceptibility of sure loss is irrational. Yet, there may be cases that it is more rational to choose small sure losses than gambling big losses. Suppose that you know if you post betting quotients of .6 on a coin landing heads and .5 on it landing tails, then the bookie will make a \$1 bet with you such that you lose \$0.1 for sure. Suppose that you also know if you post betting quotients of .5 on heads and .5 on tails, then the bookie will bet you \$1000 that the coin will land heads. If you are risk averse, you might prefer to accept the sure loss of \$0.1 to gamble on losing \$500. Hence, to you, it could be more rational to post betting quotients that violate the probability calculus [11]. The arguments have the assumption that rational agents are

Table 1: A simplified example of alternatives with possible outcomes from [18] (Table 5.4, p.156)

Alternatives	E_1	E_2	E_3
A	0	1	2
B	1	2	0
C	1	0	1
D	0	1	0
F	1	1	1
G	2	1	0

risk neutral. Nevertheless, risk neutrality is hardly observed in real-life individuals, even though the subjective probability is all about personal belief. The arguments thus instruct you that the risk averse tendency as in human nature should not be included in your personal belief. Again, you should choose the rational subjective probability that is not part of your subjective feeling.

4 Decision Making without Precise Subjective Probability

Savage [12] showed that if one's personal preferences on alternative acts and consequences satisfy the assumptions of *connectedness or partial orders*, *transitivity*, and *strong independence between preferences and states of the world*, the decision made by selecting the act that maximizes the expected utility is rational. Savage also showed how pure subjective utility and subjective probability can be determined from preferences under uncertainty.

One misconception of the expected utility theory is that decision making has to be based on the predetermined precise values of probabilities. That is, given a sigma algebra with all possible events, the subjective probability of every event should be given a unique value. Otherwise, it is impossible to make decision. In other words, unique probability values is both sufficient and necessary conditions of rational decision making. This misconception is a narrow interpretation of the expected utility theory. This section will show that unique probability values is the sufficient condition of rational decision making, but not a necessary one. Here, a simplified example from D. White's book [18] (Section 5.2.2, p.156-159) is used to illustrate that decisions can be made without unique probability values.

Using the common notations of utility theory, we denote 'x is preferred to y' by $x \succ y$, and 'x is indifferent to y' by $x \sim y$. With $u(\cdot)$ denoting the utility function, $x \succ y \Leftrightarrow u(x) > u(y)$. $x \sim y \Leftrightarrow u(x) = u(y)$. Note that in this simplified definition, it is assumed that indifference $\sim$ is transitive. This assumption is not unacceptable. Because even without such an assumption, a transitive 'equivalence' relationship ([3], p.15-18) can be derived so that this new equivalence relationship can be used instead. Six alternative actions (A, B, C, D, F, G), three possible events (E_1, E_2, E_3), and three outcomes $(0, 1, 2)$ are given in Table 1.

The preferences of a decision maker are $A \succ B$, $C \sim D$, $2 \succ 1$ and $1 \succ 0$. Let $u(0) = 0$, $u(1) = a$, and $u(2) = 1$. Therefore, $0 < a < 1$. Let the probabilities of three possible events be $P(E_1) = p_1$, $P(E_2) = p_2$, and $P(E_3) = p_3$ respectively. Based on the Savage's expected utility theory [12] (notice that he did not forbid preferences on actions, where he used a 'grand-world small-world' illustration),

$$A \succ B \Leftrightarrow E[u_A] > E[u_B] \Leftrightarrow p_2 a + p_3 > p_1 a + p_2 \tag{1}$$

$$C \sim D \Leftrightarrow E[u_C] = E[u_D] \Leftrightarrow p_1 a + p_3 a = p_2 a \tag{2}$$

Because $p_1+p_2+p_3=1$, Eq.(2) yields $p_1+p_3=p_2=1/2$. Therefore, Eq.(1) becomes $a/2+(1/2-p_1) > p_1 a + 1/2$, which leads to $0 < p_1 < a/(2a+2) < 1/4$.

Now suppose that the decision maker needs to choose one between alternatives F and G. The respective expected utilities are $E[u_F]=a$ and $E[u_G]=p_1+p_2 a$. The difference between the two is $E[u_F]-E[u_G]=a/2-p_1 > a/2-a/(2a+2)=1/(2/a+2/a^2)>0$. Therefore, without precise values of p_1 and p_3 specified, a decision of choosing F still can be made.

The above example shows that rational decisions based on the expected utility theory can still be made without the precise probability assignments. The expected utility theory is a theory of coherent preferences. Coherent preference is the main argument about rationality. The additional restriction on the values of probability measures can be regarded as a narrow interpretation of the expected utility theory. Rationality of decision making is more of preferences than of probabilities.

5 Rational Decision vs. Rational Design

As Savage wrote [12] (p.57): "According to the personalistic view, the role of the mathematical theory of probability is to enable the person using it to detect inconsistencies in his own real or envisaged behavior...". The rationality argument of decision making based on subjective probability and expected utility is about personal. How it can be applied in engineering design is still an open question. For instance, utility has limitations for engineering practices [16]. The most prominent one is the group decision making that is a common practice in the engineering design process. However, based on voting procedures, inconsistency is inevitable if the five requirements (collective rationality, unrestricted domain, Pareto principle, independence of irrelevant alternatives, and non-dictatorship) in Arrow's impossibility theorem [1] are satisfied.

The applicability of Arrow's five requirements in engineering design itself has been a debatable issue [14, 4]. Here, I do not wish to discuss it. But if design decision making is arguably all personal and should be based on Savage's expected utility theory with subjective probability, then group decision making does not exist. Ironically, the subjective probability interpretation is to promote individualism in a democratic society. Yet, in order to ensure social decisions in a group made are "rational" based on its criteria, either dictatorship is essential, or individual's are not all rational.

The main question here is what the so-called rational design is. Is rational design simply equivalent to rational decision making, as implied in [5]? Given the complexity of modern systems, hundreds or even thousands of decision makers are involved in the design process where design variables of individual components are made separately. One's decision could be made based on other decisions, or completely independently. Suppose that each decision made in the process is rational. That is, all decision makers have their own coherent preferences. This does not guarantee that the final design provides a rational engineering solution. There are two reasons. First, coherence ensures consistency but does not prevent ignorance. For instance, an engineer designs a solar power plant but holds a strong belief, with a probability of near one, that tomorrow will be the last day of which the sun will ever rise. Most likely, the resulted design is not going to be rational, even though all of his/her decisions are coherent. Sen [15] called such failure of deliberation "reflection irrationality". Second, coherence does not prevent self-interest driven manipulation. For example, an anti-society extremist does not express his true preferences or persuades fellow engineers' preferences and maneuvers pair-wise alternative voting sequences toward designing an inherently unsafe bridge. Sen [15] termed the non-correspondence between true and expressed preferences as "correspondence irrationality".

Therefore, rational decisions of a designer is not equivalent to rational design. Katsikopoulos [7] recently argued that correspondence should be used to measure the effectiveness of design, in parallel with coherence that measures personal consistency of designers. Rational design needs to satisfy both requirements of coherence and correspondence. Yet the sufficient and necessary conditions that a design is rational and the formal definition of rational design are still open questions so far.

6 Concluding Remarks

The expected utility theory and subjective probability are personal. Rationality of decision is about coherence and self-consistency internally within an individual decision maker. This premise and the associated assumptions should be kept in mind. The rationality with the expected utility theory, including the multi-attribute utility theory [16, 8], is about the coherence of *personal* preferences and nothing else. The decision maker's preference can constrain the values of subjective probabilities indirectly. Therefore, the decision maker do not have to specify precise values of probabilities directly. The philosophical arguments of Dutch book on rationality rely on various assumptions. Its applicability to real-world problems are subject to debate. Similarly, it is worthwhile to research more on the questionable argument that rational design is equivalent to rational decision making.

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