Low-carbon product design for product life cycle

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Product design stage has a profound impact on a product's carbon footprint in its life cycle. Existing low-carbon design approaches are either not able to achieve low-carbon design solutions in product life cycle or prone to a loss of optimal solutions with the consideration of product life cycle. At each stage of product life cycle, there are several alternative design solutions, which can provide extra design space and bring more opportunities for low-carbon design. In this paper, a feature-based carbon footprint element model is proposed to estimate the carbon footprint at each stage of product life cycle. A five-layer weighted directed graph-based life cycle decision space is also proposed to represent the alternative life cycle paths. The low-carbon design process is to search the design solution with the lowest carbon footprint based on the mapping between design solution space and life cycle decision space. The proposed design process is to provide an integrated approach to enumerate and combine alternative solutions at each stage of product life cycle. The low-carbon design of a cold heading machine is given as an example to demonstrate the design methodology.

Keywords: low-carbon design; carbon footprint; product design; product life cycle.

1. Introduction

The emission of greenhouse gases (GHGs), particularly carbon dioxide, is the main cause of global climate change (Hammond 2007; IPCC 2007; Oreskes 2004; U.S. GCRM 2009). Thus, the Kyoto Protocol and the Copenhagen Protocol have advised industry and governments to take actions to mitigate GHG emissions with low-carbon technology (United Nations 1998), which was defined by the Intergovernmental Panel on Climate Change (IPCC) (2007) as the technology that results in less carbon emission in the entire life cycle of a product. The term, product carbon emission or product carbon footprint, is now widely accepted as an important indicator of the environmental impact of a product, considering the emissions of GHG at all stages of the product life cycle. However, existing methods to estimate the carbon footprint are too complex and cannot be efficiently used for product design (Wiedmann and Minx 2008). Carbon footprint calculation methods targeted for low-carbon design are needed.

Low-carbon design is the design process for a product with the consideration of GHG emissions during its entire life cycle. The result of low-carbon design is the creation of products with reduced GHG emissions. The major portion of carbon footprint for a product is determined at the design stage of its life cycle. Thus, low-carbon design is an important topic in environmentally conscious product development.

One important research issue for low-carbon design is how to construct carbon footprint models in order to provide quantitative metrics for evaluation. The standards to quantitatively evaluate carbon footprint in the product life cycle are mainly ISO/TS 14067 and PAS 2050 (Gerritsen *et al.* 2011; ISO/TS 14067 2013; PAS-2050 2011). Carbon footprint is typically calculated by considering carbon emission factors and activity data (Finkbeiner 2009; PAS-2050 2011; Scipioni *et al.* 2012), which are evaluated by using the life cycle assessment (LCA) method (ISO14040 2006; Kubler *et al.* 2013; Morrison *et al.* 2013; Reap *et al.* 2008), from the raw materials acquisition stage to the manufacturing, transportation, usage, and recycle and disposal stages. LCA is based on the life cycle inventory (LCI), which is a repository that includes the data of resources and energy consumptions, and GHG emissions throughout the entire product life cycle. Current research on carbon footprint evaluation only focuses on the mapping from the bill of materials (BOM) in the LCI to carbon footprint. For instance, Jeswiet et al. (2008) proposed a carbon emission signature to connect the electrical energy used in manufacturing with the carbon emissions. Elhedhli et al. (2012) proposed a carbon footprint model in the supply chain based on Lagrangian relaxation. Ball et al. (2009)

developed a model to represent material, energy, and waste flows to support manufacturing facility design. Bocken et al. (2011) developed an eco-ideation tool to facilitate the generation of radical product and process ideas to reduce GHG emissions. Sundarakani et al. (2010) calculated carbon footprint for the supply chain. Ameta et al. (2009) developed a methodology for computing carbon footprint in the manufacturing processes from components to assemblies. Joyce et al. (2010) estimated the carbon footprint of telecommunication products. He et al. (2014) combined carbon footprint model with the consideration of data imprecision in product life cycle, which can model carbon footprint of design solutions in conceptual design.

Similar to other sustainability issues (Eddy *et al.* 2013; Gagnon *et al.* 2012), another research issue in low-carbon design is about how to reduce GHG emissions associated with a product in design decision making. Several methods of integrating carbon footprint estimation in design tools have been proposed. For instance, Song et al. (2010) developed a low-carbon product design system that integrates GHG emission data of components into BOM. Kuo (2013) developed a collaborative design framework to help enterprises collect carbon footprints and a computer-aided tool to integrate enterprises' internal systems with LCI database. Rotz et al. (2010) provided a management tool for evaluating the effects of GHG emissions and the overall carbon footprint in production systems. Pasqualino et al. (2011) studied the environmental impact of different packaging materials on the entire life cycle of products. Qi et al. (2011) proposed a dynamic configuration model that follows some modular design rules. Devanathan et al. (2010) proposed a semi-quantitative ecodesign method that is a combination of environmental life cycle assessment and visualization tools.

In summary, two basic research issues have been studied for low-carbon product design. One is how to estimate product carbon footprint quantitatively, and the other is how to enhance design procedures and decision makings that lead to carbon footprint reduction. Nevertheless, the above research efforts only focus on the relationship between LCI and carbon footprint through a mapping from BOM to carbon footprint via LCA. The alternative design solutions are compared only at the same stage of product life cycle, which, unfortunately, is likely to miss the global optimum of low-carbon design solutions across the entire product life cycle. The solution with the lowest carbon footprint at a certain stage of product life cycle is not always the one with the lowest carbon footprint from the viewpoint of the entire product life cycle. Furthermore, the details of alternative life paths for a product in its life cycle need to be captured for more accurate estimation of carbon footprint. Variations exist in the later stages of life cycle such as how products are used by different users. As a result, the above low-carbon design methods cannot provide specific guidance on how to enumerate alternatives in design solution space and choose the best design to minimize GHG emissions.

Existing approaches for low-carbon design are prone to miss the optimum solution for the product life cycle. Therefore, it is important to explore the design solution space thoroughly for potential low-carbon designs. The *design solution space* is formed by all possible parameters and configurations that provide the solutions of the design problem. The choices of parameters and configurations made for a product in the design solution space determine the major portion of carbon footprint of the product throughout its life. Yet the actual carbon footprint is the result of decisions made in its life when choosing different life paths such as transportation, usage, and recycle. All possible life paths of a product form the *life cycle decision space*. Low-carbon design methods need to depend on information in both design solution space and life cycle decision space. Evaluating all possible design alternatives with such information can improve the effectiveness of low-carbon design.

To capture the fine-grained information in life cycle decision space, in this paper, a feature-based carbon footprint element model is proposed to estimate carbon footprints for each stage of product life cycle. A five-layer weighted directed graph model is also developed to represent the life paths of a product. The low-carbon design process is then formulated as a search problem to find the shortest path in the weighted directed graph. The main contributions of this paper include a new approach to calculate life cycle carbon footprints and a new graph-based approach to help explore design alternatives at each stage of product life cycle. The graph model helps generate feasible solutions and search the optimal design with the lowest carbon footprint. It allows the designer to make decisions based on not just a single assessment of carbon footprint for one design solution, but also the variations due to the life cycle decisions made by other stakeholders such as manufacturers and users.

In this paper, the life cycle of a product is divided into five stages: raw materials acquisition, manufacturing, transportation, usage, and recycle and disposal stage. The proposed five-layer weighted directed graph model uses the above five stages to represent the life cycle decision space. A weighted graph consists of nodes, edges, and weights. In this five-layer weighted directed graph model, each stage is modelled as one layer of nodes in the life cycle decision space, and each node represents a decision made for the choices of manufacturing, transportation, etc. at a stage. Nodes are connected by directed edges, corresponding to temporal sequences of the five stages. The weight associated with each edge is the carbon footprint of the chosen path in the life cycle. With the weighted graph model, searching the optimal design with the lowest carbon footprint is then formulated as searching the shortest path.

There are three types of shortest path searching algorithms in graphs (Wang *et al.* 2005). The first one is the combinatorial traversal technique, such as label setting (Dijkstra 1959), label correcting (Ford 1956), and their hybrids (Glover *et al.* 1984). The second type is the linear programming technique, such as the primal network simplex method (Goldfarb and Jin 1999) and dual ascent method (Pallottino and Scutellà 1997). The third type is the matrix-based technique, such as Floyd-Warshall algorithms (Floyd 1962; Warshall 1962). The first two types of the shortest path searching algorithms are mainly designed to solve the single-source shortest path problem, which is the problem of computing the shortest path tree for a specific source node. The algebraic shortest path algorithms of the third type, on the other hand, are more suitable for searching the shortest paths for all pairs of nodes. The problem of searching low-carbon design in the five-layer graph-based life cycle decision space can be solved by simply applying the algorithm for single-source shortest path problem *n* times, where *n* is the size of a minimum node cover in the defined bipartite graph. However, these approaches might fail to yield feasible solutions efficiently since the exhaustive search of nodes would lead to the combinatorial explosion of solutions. Our search approach is the application of the shortest path searching algorithm developed by (Cormen *et al.* 2009). Alternative solutions at each stage of product life cycle for low-carbon design are enumerated through the five-layer graph.

In the remainder of this paper, Section 2 describes the feature-based carbon footprint model for quantitative estimation. The five-layer weighted directed graph-based life cycle decision space for the carbon footprint model and design solution space are described in Section 3. In Section 4, the searching process for low-carbon design based on the proposed models is discussed in details. The issue of finding an integrated solution with the consideration of alternative life paths at each stage of the product life cycle is addressed. In Section 5, an example of a cold heading machine is used to demonstrate the proposed low-carbon design methodology. Section 6 concludes the paper.

2. Feature-based carbon footprint model to estimate carbon footprints for product life cycle

According to the definition of product life cycle (PAS-2050 2011), the contribution of carbon footprint is classified into five stages for the entire life cycle of a product: acquisition of raw materials stage, manufacturing stage, transportation stage, usage stage, and recycle and disposal stage.

The carbon footprint of the product life cycle is defined as:

$$E_c = E_a + E_m + E_t + E_u + E_r \tag{1}$$

where E_c is the carbon footprint for the complete life cycle, and E_a, E_m, E_t, E_u, and E_r are the carbon footprints at the stages of acquisition of raw materials, manufacturing, transportation, usage, and recycle and disposal, respectively.

We define *carbon footprint feature* as a collection of important information elements associated with the low-carbon product design process, including direct and indirect carbon footprint. The feature-based *carbon footprint element model* for product life cycle proposed here is to represent all necessary information involved in the carbon footprint of a given solution for low-carbon product design. With the help of this feature-based carbon footprint element model, carbon footprint at each stage of product life cycle can be estimated quantitatively.

2.1 Feature-based carbon footprint element model for product life cycle

The carbon footprint element model at each stage is composed of carbon footprint *feature*, *relation*, and *identity*, as shown in Figure 1. Carbon footprint feature, as the necessary information involved in the calculation of carbon footprint, is composed of direct carbon footprint and indirect carbon footprint. The relation, which is used to represent the context relationship among the current carbon footprint feature and its correlated carbon footprint feature in the product life cycle, has *preceding* and *subsequent* relations based on the temporal sequence in the product life cycle. The identity, which is used to identify the carbon footprint feature, includes name and description. These elements are discussed in detail as follows:

(1) *Direct carbon footprint*: It is the GHG emissions that occur at the point of direct energy consumption during the processes of manufacturing, usage, etc. For instance, when a machine tool is used to manufacture a part, the industrial gas emission is the direct carbon footprint associated with the process.

(2) *Indirect carbon footprint*: It is the GHG emissions that are associated with the usage of energy as they occur in the situations other than the direct energy consumption, e.g. the consumption of raw materials may generate indirect carbon footprint in the mineral exploration.

(3) *Preceding relation*: the carbon footprint feature before the current carbon footprint feature in the temporal sequence of product life cycle. For instance, the carbon footprint feature at the acquisition of raw materials stage is the preceding carbon footprint feature of the corresponding carbon footprint feature at the manufacturing stage.

(4) *Subsequent relation*: the carbon footprint feature after the current carbon footprint feature in the temporal sequence of product life cycle. For instance, the carbon footprint feature at the manufacturing stage is the subsequent carbon footprint feature of the one at the acquisition of raw materials stage.

(5) *Name*: the name in the model to identify the current carbon footprint feature, such as s_{11}.

(6) *Description*: the detailed information to describe the current carbon footprint feature, such as s_{11} is a solution made by 40CrMoA steel.

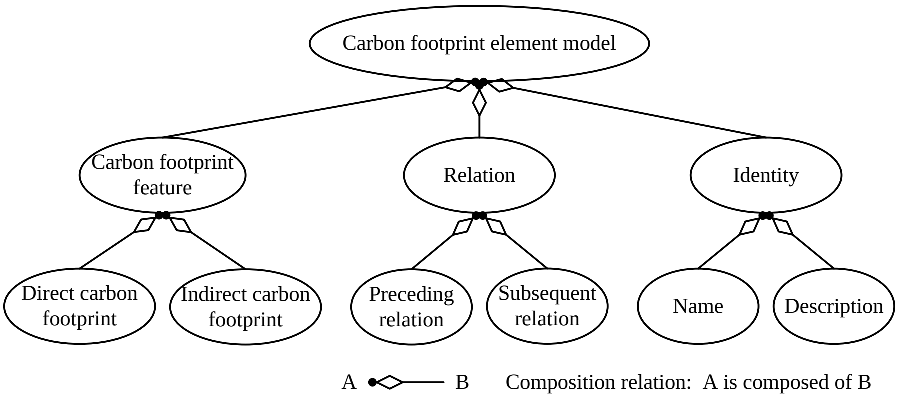

Figure 1. Carbon footprint element model for product life cycle.

2.2 Feature-based carbon footprint estimation for each stage in product life cycle

Carbon footprint at the i^{th} stage of product life cycle is generally calculated as

$$E_i = \sum_{j=1}^{M_i} M_{ij} \cdot C_{ij} + \sum_{k=1}^{N_i} G_{ik} \cdot GWP_{ik} \quad (2)$$

where the first term $M_{ij} \cdot C_{ij}$ estimates the indirect carbon footprint, whereas the second term $G_{ik} \cdot GWP_{ik}$ captures the direct carbon footprint, M_{ij} is the j^{th} activity data at the i^{th} stage of product life cycle, which is quantitative measure of activity that results in a GHG emission or removal (ISO/TS 14067 2013), C_{ij} is the carbon emission factor of the j^{th} activity at the i^{th} stage of product life cycle, G_{ik} is the direct emission of the k^{th} type of GHG at the i^{th} stage of product life cycle, and GWP_{ik} is the global warming potential of the k^{th} type GHG at the i^{th} stage of product life cycle. The index i indicates the stage symbol of the product life cycle. For instance, $i=a$ for the acquisition of raw materials stage, $i=m$ for the manufacturing stage, $i=t$ for the transportation stage, $i=u$ for the usage stage, and $i=r$ for the recycle and disposal stage, respectively. M_i is the number of activities at the i^{th} stage of product life cycle, and N_i is the total number of direct GHG emission types at the i^{th} stage of product life cycle. The carbon footprint for the acquisition of raw materials stage precedes the manufacturing stage. As an instance of Eq.(2), in the calculation of carbon footprint at the acquisition of raw materials stage E_a, M_{aj} is the consumption of the j^{th} raw material, C_{aj} is the carbon emission factor of the j^{th} raw material, G_{ak} is the emission of the k^{th} type of GHGs, GWP_{ak} is the global warming potential of the k^{th} type GHG, M_a is the number of raw material types consumed at the acquisition of raw materials stage, and N_a is the number of the direct emissions of GHGs at the acquisition of raw materials stage. The carbon footprint for the manufacturing stage is preceded by the material acquisition stage and precedes the transportation stage. As another instance of Eq.(2), in the calculation of carbon footprint at the manufacturing stage E_m, M_{mj} is the j^{th} manufacturing activity in the manufacturing process, C_{mj} is the carbon emission factors of the j^{th} manufacturing activity in manufacturing process, G_{mk} is the emissions of the k^{th} type GHGs, and GWP_{mk} is the global warming potential of the k^{th} type GHG, M_m is the number of manufacturing activities at the manufacturing stage, and N_m is the number of the direct emissions of GHGs at the manufacturing stage.

Note that some activities may only contribute small amount of GHG emissions. They will be not included in the carbon footprint calculation. In PAS 2050 (PAS-2050 2011), a rule is defined that a valid contribution from any source should result in at least 1% of the total anticipated life cycle emissions of the product.

The carbon footprint for the transportation stage is preceded by the manufacturing stage and precedes the usage stage. The carbon footprint at the transportation stage E_t is calculated as

$$E_t = \sum_{j=1}^{M_t} T_{tj} \cdot L_{tj} \cdot EI_{tj} \cdot C_{tj} + \sum_{k=1}^{N_t} G_{tk} \cdot GWP_{tk} \tag{3}$$

where T_{tj} is the quantities of the transport objects, including materials, parts, products and waste in the j^{th} transport mode, L_{tj} is the transport distance in the j^{th} transport mode, EI_{ti} is the energy intensity of the j^{th} transport mode, i.e. the energy consumption per unit of energy quantity and per unit of distance in the j^{th} transport mode, C_{tj} is the carbon emission factor of energy consumption in the j^{th} transport mode, G_{tk} is the emission of the k^{th} GHG at the transportation stage, and GWP_{tk} is the global warming potential of the k^{th} GHG, M_t is the number of transport modes at the transportation stage, and N_t is the number of the direct emissions of GHGs at the transportation stage.

The carbon footprint for the usage stage is preceded by the transportation stage and precedes the recycle and disposal stage. At the usage stage, there are three kinds of activities, which are activities in the normal use and inspection process, in the re-manufacturing process of repaired component, and in the re-assembly process of repaired components. The carbon footprint at the usage stage E_u is calculated as

$$E_u = \sum_{j=1}^{M_{u1}} U_{uj} \cdot C_{uj} + \sum_{j=1}^{M_{u2}} M_{uj} \cdot \frac{L}{L_{uj}} \cdot C_{mj} + \sum_{j=1}^{M_{u3}} F_j \cdot \frac{L}{L_{uj}} \cdot C_{fj} + \sum_{k=1}^{N_u} G_{uk} \cdot GWP_{uk} \tag{4}$$

where U_{uj} is the activity data in the j^{th} normal use and inspection, C_{uj} is the corresponding carbon emission factor , M_{uj} is the activity data in the j^{th} manufacturing process of repaired components, C_{mj} is the corresponding carbon emission factor, F_{uj} is the activity data in the j^{th} assembly activity of repaired components, C_{fj} is the corresponding carbon emission factor, L is the service life of the product, L_j is the service life of the j^{th} component, G_{uk} is the emission of the k^{th} type GHGs at the usage stage, GWP_{uk} is the global warming potential of the k^{th} type of GHGs, M_{u1} is the number of activities in normal use and inspection at the usage stage, M_{u2} is the number of activities in the manufacturing process of repaired component at the usage stage, M_{u3} is the number of activities in the assembly activity of repaired component at the usage stage, and N_u is the number of the direct emissions of GHGs at the usage stage.

The carbon footprint for the recycle and disposal stage is preceded by the usage stage. At this stage, there are two kinds of activities, activities in the disposal process and activities in recycle process. The carbon footprint at the recycle and disposal stage E_r is calculated as

$$E_r = \sum_{j=1}^{M_{r1}} D_{rj} \cdot C_{dj} + \sum_{j=1}^{M_{r2}} (R_{rj} \cdot C_{rj} - G_{rj}) + \sum_{k=1}^{N_r} G_{rk} \cdot GWP_k \qquad (5)$$

where D_{rj} is the quantity of the j^{th} activity or component in the product disposal process, C_{dj} is the corresponding carbon emission factor for each of the j^{th} activity, R_{rj} is the quantity of the j^{th} activity in the reusing process, C_{rj} is the corresponding carbon emission factor for the j^{th} activity, G_{rj} is the carbon emission reduced in the reusing process of the j^{th} activity, G_{rk} is the emission of the k^{th} type of GHGs in the recycle and disposal stage, GWP_{rk} is the global warming potential of the k^{th} type of GHGs, M_{r1} is the number of activities or components in the product disposal process, M_{r2} is the number of activities in the reusing process, and N_r is the number of the direct GHG emission types at the recycle and disposal stage.

2.3 Calculation process of product carbon footprint

Based on the ISO standards (ISO/TS 14067 2013; PAS-2050 2011), the five steps to calculate the product carbon footprint are:

Step 1. Identify all materials, activities and processes that contribute to the chosen product's life cycle, including all material, energy and waste flows.

Step 2. Confirm system boundary to define the scope for the product carbon footprint.

Step 3. Collect all the data on the amount of material, activities and emission factors across all life cycle stages.

Step 4. Calculate the product carbon footprint at each stage of product life cycle using Eq. (2-5), respectively, and then obtain the whole carbon footprint of the entire product life cycle using Eq. (1).

Step 5. Assess precision of the carbon footprint analysis if necessary.

3. Graph-based life cycle model for low-carbon design

3.1 Definitions

In this section, a graph-based representation for carbon footprint element model is presented. Figure 2 shows the basic elements of this model which are defined as follows.

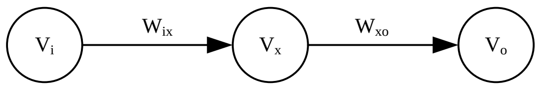

Figure 2. Graph-based representation for carbon footprint element model.

(1) *Node*: A node, such as nodes V_i, V_x, and V_o in Figure 2, represents a choice for life cycle decisions that stakeholders make at one of the five stages in the product life cycle.

(2) *Directed edge*: A directed edge links two nodes, when there exists a sequential relationship between the two nodes.

The nodes are classified into *preceding node*, *current node*, and *subsequent node*.

(3) *Current node*: Current node represents a decision at the current stage of product life cycle.

(4) *Preceding node*: A preceding node with respect to the current node is an adjacent one that has an edge connecting to the current node. It represents a decision at the previous stage.

(5) *Subsequent node*: A subsequent node with respect to the current node is an adjacent one that the current node connects to. It represents a decision at the subsequent stage.

A current node may not have a preceding node, e.g. the node at the acquisition of raw materials stage. It is also possible that a node does not have a preceding one because it is not supported by the subsequent stages. For instance, with some initial choices of new materials, existing manufacturing techniques have difficulties to process these materials. Obviously the current node at the recycle and disposal stage has no subsequent node.

(6) *Weight*: The value of a weight associated with an edge, such as w_{ix} and w_{xo} in Figure 2, represents the carbon footprint of the chosen path in the life cycle. As the node at the recycle and disposal stage has no subsequent nodes, the corresponding carbon footprint at this stage is merged into the carbon footprint at the usage stage.

As an example of the general model in Figure 2, if the current node V_x represents precision grind at the manufacturing stage, one of the alternative materials at the acquisition of raw materials stage is steel with weight w_{ix}, which corresponds to the carbon footprint of the material during materials acquisition stage. V_o denotes air transportation if it is chosen at the transportation stage. The weight w_{xo} corresponds to the carbon footprint of the precision grind process.

3.2 Mapping between design solution space and life cycle decision space

As mentioned in Section 1, information in two spaces is used in low-carbon product design. One is the design solution space, and the other is the life cycle decision space. The design solutions with product related parameters form the design solution space where the designer makes decisions, whereas the choices of life cycle decisions on material selection, manufacturing, transportation, usage, and recycle form the life cycle decision space where other stakeholders make decisions. The combination of decisions at each stage in the whole product life cycle forms a life cycle path of a design solution. There could be several design solutions to meet the design requirements in the design solution space, from which the designer can choose. One design solution could correspond to several life cycle decisions from the manufacturing, transportation, usage, to the recycle and disposal stage, which have different carbon footprints. For instance, the same machine tool might be used with different working frequencies and conditions, which results in different carbon footprints. The life cycle path of a design solution is the result of the combined decisions from all stakeholders that are involved in the entire product life cycle, including the designer, manufacturer, transportation service provider, customer, etc. Unfortunately, the designer only makes decisions in the design solution space and cannot determine the entire life cycle path of a product. In the life cycle decision space, the choices of life cycle path for a design solution are also made by other stakeholders. The designer can only make the best design decisions based on the available information in the life cycle decision space. Thus, low-carbon product design is a decision making process that relies on the mapping between the design solution space and the life cycle decision space.

In the design solution space, there might be different design concepts and solutions, denoted as $DS=\{DS_1, DS_2, \dots , DS_d\}$, which satisfy the design requirements. Each design solution is assumed to have a unique choice at the raw materials stage and the manufacturing stage. In the life cycle decision space, choices are made from several alternatives at each stage, which form the life cycle path. For the acquisition of raw materials stage, the set of alternatives is denoted as $S_1=\{s_{11}, s_{12}, \dots, s_{1i},\dots, s_{1a}\}$ where s_{1i} (i=1,2,…,a) is an alternative corresponding to one selection of materials. At the manufacturing stage, the set of alternatives is $S_2=\{s_{21}, s_{22},\dots, s_{2i},\dots, s_{2m}\}$ with s_{2i}

(i=1,2,…,m) represents a combination of manufacturing processes to realize the product. Similarly, the set of transportation alternatives is denoted as $S_3=\{s_{31}, s_{32}, \ldots, s_{3i},\ldots, s_{3t}\}$, the set of different conditions of use during the usage stage is denoted as $S_4=\{s_{41}, s_{42}, \ldots, s_{4i}, \ldots, s_{4u}\}$, and the set of different recycle and disposal methods is denoted as $S_5=\{s_{51}, s_{52}, \ldots, s_{5i},\ldots, s_{5r}\}$. The weighted directed graph models the different alternatives of life cycle paths. A life cycle path is a sequence of nodes. The life cycle decision space is formed by all possible life cycle paths of the product. The materials and manufacturing methods are selected from the life cycle decision space. The choice of a design solution is also based on the decisions made at the subsequent stages in the life cycle decision space.

The relationships between the life cycle paths in the life cycle decision space and the design solutions in the design solution space are illustrated in Figure 3. After a life cycle path in the life cycle decision space is mapped to the design solution space, a design solution is obtained. It is also possible that one design solution corresponds to several life cycle paths in the life cycle decision space. For instance, in Figure 3, a design solution DS_1 can be mapped to the life cycle path $s_{11}\rightarrow s_{22}\rightarrow s_{31}\rightarrow s_{41}\rightarrow s_{52}$, or the life cycle path $s_{11}\rightarrow s_{22}\rightarrow s_{34}\rightarrow s_{4j}\rightarrow s_{5u}$ in the life cycle decision space, and DS_d can be mapped to the life cycle path $s_{12}\rightarrow s_{23}\rightarrow s_{34}\rightarrow s_{4j}\rightarrow s_{5s}$. The objective of low-carbon product design is to find the optimum design solution, which corresponds to the life cycle path that has the minimum GHG impact among the possible ones.

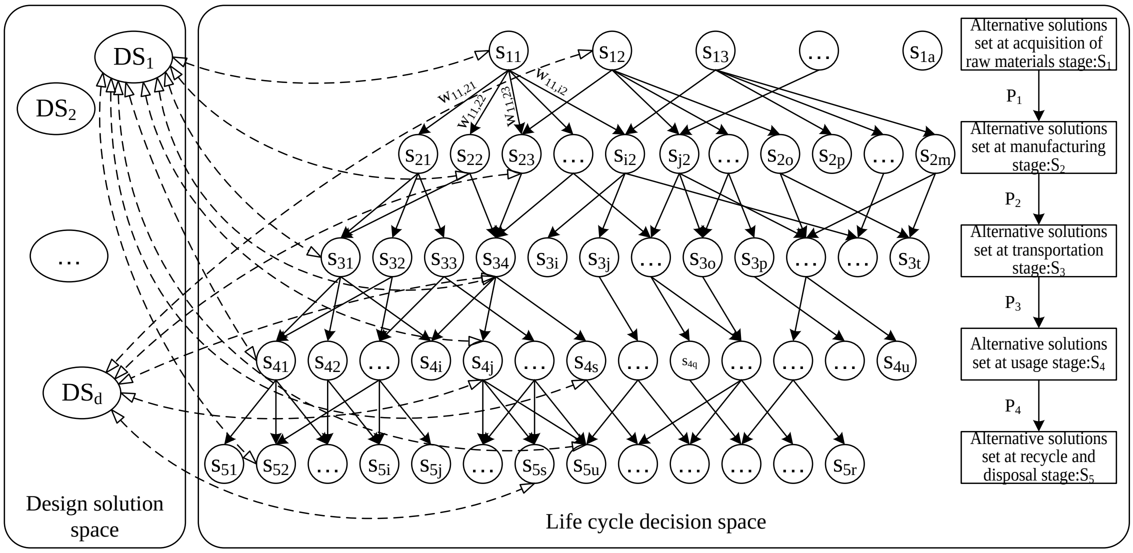

Figure 3. The mapping between design solution space and life cycle decision space for low-carbon product design.

In Figure 3, the sequences P_1, P_2, P_3, and P_4, are the binary relations of S_1 to S_2, S_2 to S_3, S_3 to S_4, and S_4 to S_5, respectively, and are defined as

$$P_{\mathrm{i}}=\{< s_{ip}, s_{(i+1)q} >|\ s_{ip} \in S_i, s_{(i+1)q} \in S_{i+1}\} \tag{6}$$

(i=1,2,3,4; p=1,2,…,$|S_i|$; q=1,2,…,$|S_{i+1}|$)

The set of life cycle paths R can be defined as:

$$\begin{aligned} R = \{ &< s_{1p}, s_{5q} >| s_{1p} \in S_1, s_{5q} \in S_5, \exists s_{2i} \in S_2, \exists s_{3j} \in S_3, \exists s_{4k} \in S_4, \\ &< s_{1p}, s_{2i} > \in P_1, < s_{2i}, s_{3j} > \in P_2, < s_{3j}, s_{4k} > \in P_3, < s_{4k}, s_{5q} > \in P_4 \} \end{aligned} \tag{7}$$

(i, j, k, p=1,2,…,$|S_1|$; q=1,2,…,$|S_5|$)

As shown in Figure 3, several mappings exist between the design solution space and the life cycle decision space. The combination of materials and manufacturing methods determines the possible design solutions. Design solutions differ from each other with either different materials or different manufacturing methods. After the materials and the subsequent manufacturing methods are chosen in the life cycle decision space, a design solution is then determined by the mapping from the life cycle decision space to the design solution space. Similarly, after one design solution is chosen in the design solution space, its materials and the corresponding manufacturing methods are uniquely obtained. The life cycle paths are then traversed for the transportation, usage, and recycle and disposal stages. In this way, the one-to-many mapping from the design solution space to the life cycle decision space is obtained.

4. Low-carbon product design for product life cycle through weighted directed graph-based life cycle decision space

In the life cycle decision space in Figure 3, a weighted five-layer directed graph $G(V, E)$ with the collections of nodes V and edges E is given with a weight function $w_{(ij)(pq)}$ that maps each directed edge (s_{ij},s_{pq}) between two adjacent nodes s_{ij} and s_{pq} to a real-valued weight, which is the carbon footprint of the corresponding alternative sequence at each stage in the product life cycle. The total carbon footprint of a life cycle path is the sum of all weights along the path. The shortest path (a path with the minimum weight) from s_{ij} to s_{pq} is found by searching through every pair of nodes s_{ij} and s_{pq}. Given that one design solution could be mapped to multiple life cycle paths because of different transportation, use, and recycling methods, two types of low-carbon solutions can be chosen in design practice. One is an ideal solution, and the other is a practical solution. An *ideal design solution* is the one with the lowest total carbon footprint corresponding to the lowest possible life cycle path in the one-to-many mapping relationship. A *practical design solution* is the one with the lowest expected value of total carbon footprint from all possible life cycle paths in the one-to-many mapping relationship. The purpose of the proposed approach is to explore the possible life cycle paths and select the best for the life cycle. Given that the decisions made by other stakeholders in the later stages of product life are out of the control of the designer, the ideal design solution may not be robust. Therefore, the practical design solution is also introduced. Design optimization in the design solution space can be performed based on the information provided in the life cycle decision space. The key point of the proposed approach is to explore and optimize the life cycle paths selected during the life cycle. Product design itself in the design solution space can also be optimized in the product design process.

The algorithm to calculate the carbon footprint is described as follows:

Step 1. Characterize the structure of the solution.

An intermediate vertex of a path p=<s_{1j},s_{2p},...,s_{5l}> is a node in p other than s_{1j} and s_{5l}, i.e. any node in the set {s_{2k},s_{3p},...,s_{4q}}. The vertices of G are given as V={1,2,...,n}. Let us consider a subset {1,2,...,k} of vertices for some k. For any pair of vertices $i, j \in V$, consider all paths from i to j whose intermediate vertices are drawn from {1,2,...,k}, and let p be the path with the minimum weight among them. The weight of the shortest path p from s_{ij} to s_{pq} for which all intermediate nodes are in the set {1,2,...,k} is

$$d_{(ij,pq)}^{(k)} = \min(d_{(ij,pq)}^{(k-1)}, d_{(ij,k)}^{(k-1)} + d_{(k,pq)}^{(k-1)}) \quad (8)$$

Step 2. Compute the weight matrix W.

For convenience, we assume that the nodes are numbered as 1i, 2j,…,|V|, so that the input is an $n\times n$ matrix W, representing the weights of an n-node directed graph G=(V, E). That is, $W = (w_{ij,pq})$, where

$$w_{ij,pq} = \begin{cases} 0 & (\,if\ ij = pq); \\ the\ weight\ of\ edge(ij, pq) & (if\ ij \neq pq\ and (ij, pq) \in E); \\ \infty & (if\ ij \neq pq\ and (ij, pq) \notin E). \end{cases} \quad (9)$$

Step 3. Define the value of an optimal solution recursively.

Based on Eqs.(10) and (11), the weight of the shortest path $d_{(ij,pq)}^{(k)}$ from node ij to node pq is calculated as

$$d_{(ij,pq)}^{(k)} = \begin{cases} w_{ij,pq} & (if \quad k=0) \\ \min(d_{(ij,pq)}^{(k-1)}, d_{(ij,k)}^{(k-1)} + d_{(k,pq)}^{(k-1)}) & (if \quad k \geq 1) \end{cases} \tag{10}$$

for which all intermediate nodes are in the set {1,2,…,*k*}. When *k*=0, a path from node *ij* to node *pq* without intermediate node numbered higher than 0 has no intermediate nodes at all. Such a path has at most one edge, and hence $d_{(ij,pq)}^{(0)} = w_{ij,pq}$.

For any path, all intermediate nodes are in set {1,2,…,*n*}, the matrix is

$$D^n = (d_{ij,pq}^n) \tag{11}$$

Step 4. Compute the total weight of the shortest path from bottom to top.

Eq.(10) is used to compute the values of $d_{(ij,pq)}^{(k)}$ in the order of increasing values of *k*. Its input is an $n \times n$ matrix *W* defined as in Eq.(9). The procedure returns the matrix $D^{(n)}$ of the shortest path.

Step 5. Construct the shortest paths.

To compute the predecessor matrix *Π*, in which its element $\pi_{ij,pq}$ is the predecessor of node *pq* on a shortest path from node *ij*. Compute a series of matrices $\Pi^{(0)}$, $\Pi^{(1)}$, $\Pi^{(2)}$, …, $\Pi^{(n)}$ sequentially, where element $\pi_{ij,pq}^{(k)}$ is defined as the predecessor of node *pq* on the shortest path from vertex *ij* with all intermediate nodes in {1,2,...,*k*}, $\Pi=\Pi^{(n)}$.

The element $\pi_{ij,pq}^{(k)}$ of predecessor matrix *Π*:

Case (1): When *k*=0, the shortest path from *i* to *j* has no intermediate nodes, thus

$$\pi_{ij,pq}^{(k)} = \begin{cases} \text{NULL} & (\text{if } ij = pq \text{ and } w_{ij,pq} = \infty); \\ ij & (\text{if } ij \neq pq \text{ and } w_{ij,pq} < \infty). \end{cases} \tag{14}$$

Case (2): When *k*≥1, the shortest path from *ij* to *pq* has intermediate nodes, thus

$$\pi_{ij,pq}^{(k)} = \begin{cases} \pi_{ij,pq}^{(k-1)} & (\text{if } d_{ij,pq}^{(k-1)} \leq d_{ij,k}^{(k-1)} + d_{k,pq}^{(k-1)}); \\ \pi_{k,pq}^{(k-1)} & (\text{if } d_{ij,pq}^{(k-1)} > d_{ij,k}^{(k-1)} + d_{k,pq}^{(k-1)}). \end{cases} \tag{15}$$

Step 6. Find all of the reachable paths with Π in the descending order.

Using the final predecessor matrix $\Pi=\Pi^{(n)}$, the following recursive procedure finds the shortest path between nodes *ij* and *pq*. It discovers every node $v \in V$ that is reachable from the source *s*. Moreover, for any vertex $v \neq s$ that is reachable from *s*, one of the shortest paths from *s* to *v* is the shortest path from *s* to the preceding node of *v* followed by the edge from the preceding node to node *v*.

Step 7. Obtain the ideal low-carbon design solution.

Take the node at the acquisition of raw materials stage as the initial node, and the node at the recycle and disposal stage as the terminal node. Enumerate all the combinations between the initial and terminal nodes. The best solution is the one with the minimum weight along the life cycle path. After mapping the path from the life cycle decision space to the design solution space, the ideal design solution with the lowest carbon footprint is obtained.

Step 8. Calculate the expected value of carbon footprint for a design solution that corresponds to multiple life cycle paths and obtain the practical low-carbon design solution that has the lowest expected value of carbon footprint.

Multiple life cycle paths with different carbon footprints may be mapped to one design solution because different options of transportation, use, and recycle. A probabilistic approach is taken to calculate the expected value of carbon footprint for such design solution. For a design solution that corresponds to *l* life cycle paths in the life cycle decision space, if each life cycle path has a carbon footprint E_{ci} and the probability that the product's life follows the i^{th} path is p_i ($\sum_{i=1}^{l} p_i = 1$), then the expected value of the design solution is

$$\mathbb{E}(E_c) = \sum_{i=1}^{l} E_{ci} p_i \tag{14}$$

The design solution with the lowest expected value of carbon footprint is then taken as the practical low-carbon design solution.

The most concerned issues in low-carbon product design are the effectiveness of design results and efficiency of the design process. The proposed graph-based approach captures and explores design alternatives at each stage of product life cycle, which generates feasible solutions and searches the optimal design with the lowest carbon footprint. It allows design decisions to include the consideration of life cycle decisions. The proposed methodology can also suggest feasible design alternatives across life cycle. The traditional low-carbon design methods could only obtain local optimization without the consideration of the entire product life cycle. The proposed methodology in this paper could obtain global optimization of the lowest carbon footprint with the consideration of the entire product life cycle. The carbon footprint of the design solutions obtained from our proposed method is less than those obtained from other low-carbon design methods, which had no consideration of the entire product life cycle. Therefore, our method is more effective than the existing ones.

In the proposed method, searching the low-carbon design in the five-layer graph-based life cycle decision space can be done by simply applying the single source shortest path algorithm z times, where z is the size of the minimum node cover on the five-layer directed graph. This approach could be more efficient for the problems with small z than the proposed algorithm. However, when z is large, the repetition of single shortest path algorithm may involve more computation than necessary. For example, suppose that N_a, N_m, N_t, N_u, and N_r are the total number of nodes respectively for the five lifecycle stages. Each node needs to appear exactly once in the source and destination node set. The single shortest path algorithm needs to be applied

$N_a \times N_m \times N_t \times N_u \times N_r$ times, where the computational effort for searching the paths that correspond to some larger carbon footprints than we previously had is unnecessary. The algorithm applied in this paper is more suitable for five-layer directed graphs. The execution time of the algorithm is related to the triply nested loops. Therefore, the time complexity of the algorithm is $O(N^3)$ where $N=N_a+N_m+N_t+N_u+N_r$ is the total number of nodes. If some life cycle choices do not have subsequent nodes, then the computation of carbon footprint in the life path is not necessary. Our algorithm terminates when all life cycle paths that potentially have a lower carbon footprint than the best one found so far are traversed, and the shortest path is returned.

5. Applications

As a case study, a cold heading machine is given as an example of low-carbon design to illustrate and verify the feasibility of the proposed low-carbon design approach. Cold heading is a process that uses die forms and punches to produce parts from metal wire, in which a force driven by a punch is applied to push materials through a die into a new shape.

The life cycle decisions at the five stages of the life cycle of the cold heading machine are denoted as node sets S_1, S_2, S_3, S_4, and S_5, respectively. Among these five stages, each stage has different choices for life cycle decisions, which might also arise from different conceptual design solutions in the design solution space. For instance, $S_1=\{s_{11}, s_{12}, s_{13}, \ldots, s_{1m}\}$ is used to represent various choices at the acquisition of raw materials stage, and $S_2=\{s_{21}, s_{22}, s_{23}, \ldots, s_{2n}\}$, $S_3=\{s_{31}, s_{32}, s_{33}, \ldots, s_{3p}\}$, $S_4=\{s_{41}, s_{42}, s_{43}, \ldots, s_{4q}\}$, and $S_5=\{s_{51}, s_{52}, s_{53}, \ldots, s_{5r}\}$ are used to represent the choices at the manufacturing, transportation, usage and recycle and disposal stage, respectively. Tables 1 and 2 show some examples of choices at the raw material acquisition and manufacturing stages for the cold heading machine, respectively. Two examples, choices 1 and 2, are explained in Table 3.

Table 1. Example life cycle choices at the acquisition of raw materials stage.

Node	Choice
s_{11}	Machine body (HT200), Sliding table (QT500), Crankshaft (45CrMo), Transmission gear and transmission spur gear (45), Flywheel (HT250), etc.
s_{12}	Machine body (Resin concrete), Sliding table (Metal, plastic composite materials), Crankshaft (45CrMo), Transmission gear and transmission spur gear (POM), Flywheel (HT250), etc.
s_{13}	Machine body (Resin concrete), Sliding table (Metal, plastic composite materials), Crankshaft (45CrMo), Transmission gear and transmission spur gear (20Cr), Flywheel (20Cr), etc.
s_{14}	Machine body (QT600), Sliding table (QT500), Crankshaft (45CrMo), Transmission gear and transmission spur gear (45), Flywheel (HT250), etc.
s_{15}	Machine body (HT200), Sliding table (QT500), Crankshaft (45), Transmission gear and transmission spur gear (45), Flywheel (HT250), etc.
s_{16}	Machine body (Resin concrete), Sliding table (Metal, plastic composite materials), Crankshaft (45CrMo), Transmission gear and transmission spur gear (POM), Flywheel (20Cr), etc.
s_{17}	Machine body (QT600), Sliding table (T8), Crankshaft (45CrMo), Transmission gear and transmission spur gear (45), Flywheel (HT250), etc.
s_{18}	Machine body (HT200), Sliding table (QT500), Crankshaft (QT600), Transmission gear and transmission spur gear (45), Flywheel (HT250), etc.
s_{19}	Machine body (Resin concrete), Sliding table (Metal, plastic composite materials), Crankshaft (45CrMo), Transmission gear and Transmission spur gear (POM), Flywheel (20Cr), etc.

Table 2. Example life cycle choices at the manufacturing stage.

Node	Choice
s_{21}	Machine body (Choice1), Sliding table (Choice1), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice1). The whole machine assembled manually.
s_{22}	Machine body (Choice1), Sliding table (Choice1), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice1). The whole machine assembled by robots.

s_{23}	Machine body (Choice2), Sliding table (Choice4), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice2). The whole machine assembled manually.
s_{24}	Machine body (Choice2), Sliding table (Choice4), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice2), Flywheel (Choice1). The whole machine assembled manually.
s_{25}	Machine body (Choice1), Sliding table (Choice2), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice1). The whole machine assembled manually.
s_{26}	Machine body (Choice1), Sliding table (Choice3), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice1). The whole machine assembled manually.
s_{27}	Machine body (Choice2), Sliding table (Choice4), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice2). The whole machine assembled by robots.
s_{28}	Machine body (Choice1), Sliding table (Choice3), Crankshaft (Choice1), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice1). The whole machine assembled by robots.
s_{29}	Machine body (Choice2), Sliding table (Choice4), Crankshaft (Choice2), Transmission gear and transmission spur gear (Choice2), Flywheel (Choice1). The whole machine assembled manually.
s_{210}	Machine body (Choice1), Sliding table (Choice1), Crankshaft (Choice2), Transmission gear and transmission spur gear (Choice1), Flywheel (Choice1). The whole machine assembled manually.

Table 3. Illustration of life cycle choices in Table 2.

Part name	No.	Choices at the manufacturing stage
Machine body	Choice1	Casting→Clean, sand blast, then paint base coat→Ruling→ Foreplane lathe bed bottom →Rough mill every guideway surface and joint surfaces→Artificial aging treatment→Try plane undersurface→ Semi-finish mill guideway surface, facies medialis, joint surfaces, facies lateralis of the lathe bed→Heat treatment→Finish plane undersurface→ Drill hole→final inspect.
	Choice2	The Resin concrete mixture which is constituted by fluid resin, diluent, curing agent and aggregate filler, through knock outing, loading, stiring and vibration molding. The resin concrete mixture will be made into finished product.

Sliding table	Choice1	Production arrangements→Modelling and core→Pouring molten→ Cleaning→The first tempering→Rough finish→The second tempering →Finish-milling(except the guideway surface)→ Inductively hardened →Accurate grinding guideway surface.
	Choice2	Forge, normalize→Foreplane all-round face→Coarse grind all-round face→Mill→Grind→Carburize→Stroke drill clamp→Quench, straighten, cold quenching→Coarse grinding→Magnetic inspect→Low tempering→Medium grind→Low tempering→ Accurate grinding.
	Choice3	Spheroidizing annealing→Plane the guideway surface →Mill→ Coarse grind all-round face →Supersonic frequency quench→ Medium grind→Low temperature age→Accurate grind→Low temperature age→Supergrind.
	Choice4	Production arrangements→Modelling and core→Pouring molten→ Cleaning→The first tempering→Rough finish→The second tempering →Finish-milling (except the guideway surface)→Accurate grinding guideway surface→Clean→Plastic coated→Press hardening.
Crankshaft	Choice1	Forge(normalize)→Rough finish→normalize→Hardening and tempering→Finish machining→Final inspect.
	Choice2	Cast, clean→Normalize→Rough surfaces on both sides→Rough turning on both ends of main journal→Rough shaft diameter of rod→Finish turning on both ends of main journal→Finish turning shaft diameter of rod→Finish turning surfaces on both sides→Final inspection.
Transmission gear	Choice1	Cutting→forging blank→Normalizing→Rough→ Finish turning→ Hobbing→Other processing→Deburring by fitter→Tooth surface quenching→Tempering→Grind→Gear grinding→Final inspection.
	Choice2	Injection molding
Flywheel	Choice1	Forging→Cleaning→Artificial aging→Fine cleaning→Non-machined surfaces coated with rust→Lathe→Slotting the keyway→ Drill→Static balance check→ Final inspection.
	Choice2	Cutting→Blank quality inspection→Heating→Forming→Pre-forging →Finish-forging→Cutting off→Trimming and punching→Surface cleaning→Correction→Coining→Tempering→Warehousing.

At the transportation stage, the machine is transported from Ningbo City to Shanghai City. The machine can be transported via highway, or the combination of highway and railway, ship or air. The highway transportation is denoted as node s_{31}, highway and railway, highway and ship, highway and air are denoted as s_{32}, s_{33} and s_{34}, respectively, as shown in Table 4.

Table 4. Life cycle choices at the transportation stage.

Node	Choice
s_{31}	The machine is transported by highway.
s_{32}	The machine is transported by highway and railway.
s_{33}	The machine is transported by highway and ship.
s_{34}	The machine is transported by highway and air.

At the usage stage, the carbon footprint includes the direct carbon footprint and the indirect carbon footprint in the installation, inspection, normal use and repair process. The service life of the cold heading machine is 15 years, and the working time is 300 days per year. There are two possible choices at this stage, shown in Table 5.

Table 5. Life cycle choices at the usage stage.

Node	Choice
s_{41}	Cold heading machine runs 12 hours a day efficiently. According to the processing of materials, size, etc., production speed is manually controlled, and the production speed is slow. 10 times a year during the usage stage is scheduled for inspection. Parts are replaced when 80% of them are damaged. The machine is repaired 8 times a year.
s_{42}	Cold heading machine runs 10 hours a day efficiently. Automatic frequency control system is used at the usage stage to achieve stepless speed, and the production speed is fast. 20 times a year during the usage stage is scheduled for inspection. Parts are replaced when 50% of them are damaged. The machine is repaired 12 times a year.

Compared to the previous four stages, the recycling process at the recycle and disposal stage reduces carbon footprint, in addition to the energy consumption and GHGs emissions, as shown in Table 6.

Table 6. Life cycle choices at the recycle and disposal stage.

Node	Choice
s_{51}	The materials of the machine are recycled.

s_{52}	The materials of the machine are remanufactured.

The carbon footprints are calculated with the model proposed in Section 2.2, as shown in Table 7.

Table 7. All the weights (carbon footprint, Unit: kg CO_{2e}) between every two adjacent nodes with directed edge.

s_{ij}	s_{pq}	Weights	s_{ij}	s_{pq}	Weights	s_{ij}	s_{pq}	Weights	s_{ij}	s_{pq}	Weights
s_{11}	s_{21}	9,550	s_{17}	s_{26}	9,650	s_{25}	s_{32}	8,100	s_{32}	s_{42}	308
s_{11}	s_{22}	9,550	s_{17}	s_{28}	9,650	s_{26}	s_{31}	7,290	s_{33}	s_{41}	368
s_{12}	s_{24}	8,618	s_{18}	s_{210}	9,463	s_{26}	s_{32}	7,290	s_{33}	s_{42}	368
s_{12}	s_{29}	8,618	s_{19}	s_{24}	8,846	s_{28}	s_{34}	7,776	s_{34}	s_{41}	2,682
s_{13}	s_{23}	9,132	s_{19}	s_{29}	8,846	s_{29}	s_{31}	5,670	s_{34}	s_{42}	2,682
s_{13}	s_{27}	9,132	s_{21}	s_{31}	7,290	s_{29}	s_{33}	5,670	s_{41}	s_{51}	655,170
s_{14}	s_{21}	9,600	s_{21}	s_{32}	7,290	s_{210}	s_{31}	6,885	s_{41}	s_{52}	654,879
s_{14}	s_{22}	9,600	s_{22}	s_{34}	7,776	s_{210}	s_{32}	6,885	s_{42}	s_{51}	511,557
s_{15}	s_{21}	9,500	s_{24}	s_{31}	6,075	s_{31}	s_{41}	810	s_{42}	s_{52}	511,266
s_{15}	s_{22}	9,500	s_{24}	s_{33}	6,075	s_{31}	s_{42}	810			
s_{17}	s_{25}	9,650	s_{25}	s_{31}	8,100	s_{32}	s_{41}	308			

According to Table 7, the weighted directed graph-based life cycle decision space and the corresponding design solution space for low-carbon cold heading machine design using the method proposed in Section 3 is shown in Figure 4.

The shortest path search algorithm in Section 4 is implemented in programming language C. The shortest path is found as $s_{12} \rightarrow s_{29} \rightarrow s_{33} \rightarrow s_{42} \rightarrow s_{52}$. The corresponding design solution is taken as the ideal design solution for the cold heading machine, with the carbon footprint value of 525,922 kg CO_{2e}.

For each design solution, the expected value of carbon footprints among its alternative life cycle paths is also calculated, shown in Table 8, assuming all life cycle paths have the same probability in the calculation. For example, the design solution DS_1 has eight equally probable life cycle paths and an expected carbon footprint value of

601,536.125 kg CO_{2e}. The lowest expected value of carbon footprint is 598,060 kg CO_{2e} corresponding to design solution DS_6, which has eight life cycle paths. DS_6 is then the practical design solution for the low-carbon design.

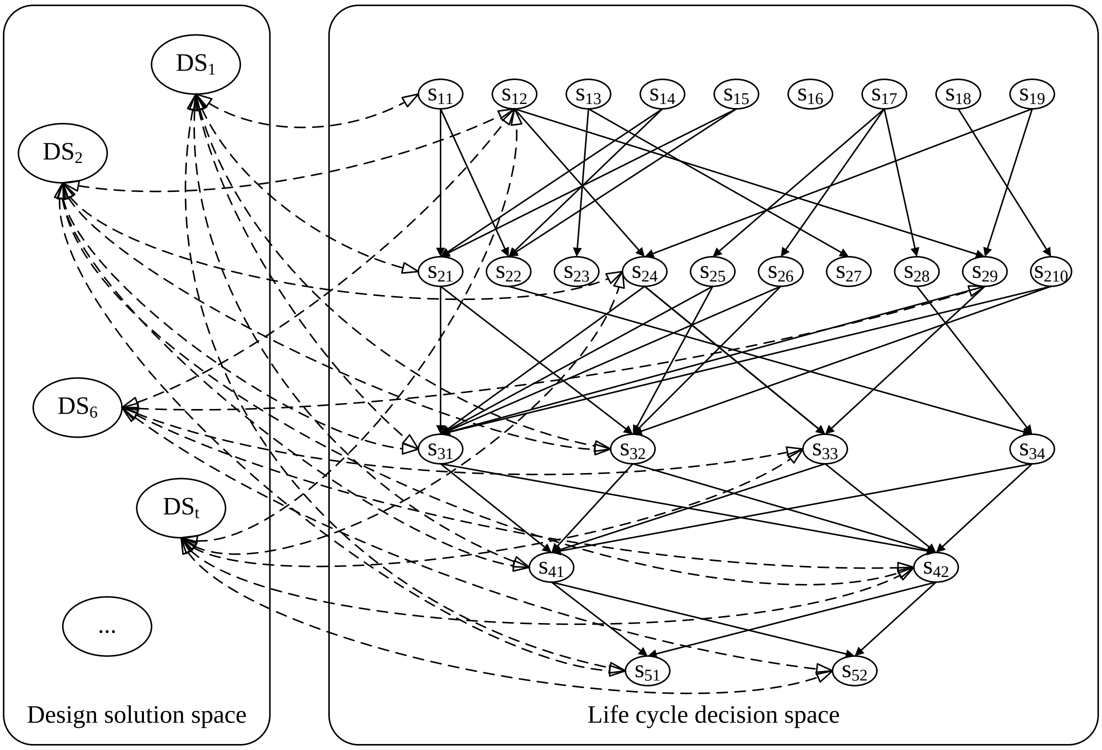

Figure 4. Examples of search results of low-carbon design for the cold heading machine.

Table 8. Examples of carbon footprint of design solutions.

Design solution	Life cycle path	Carbon footprint (kg CO_{2e})	Expected value (kg CO_{2e})
DS_1	$s_{11} \rightarrow s_{21} \rightarrow s_{31} \rightarrow s_{41} \rightarrow s_{51}$	672,820	601,536.125
	$s_{11} \rightarrow s_{21} \rightarrow s_{31} \rightarrow s_{42} \rightarrow s_{51}$	529,270	
	$s_{11} \rightarrow s_{21} \rightarrow s_{31} \rightarrow s_{41} \rightarrow s_{52}$	672,529	
	$s_{11} \rightarrow s_{21} \rightarrow s_{31} \rightarrow s_{42} \rightarrow s_{52}$	528,916	
	$s_{11} \rightarrow s_{21} \rightarrow s_{32} \rightarrow s_{41} \rightarrow s_{51}$	672,318	
	$s_{11} \rightarrow s_{21} \rightarrow s_{32} \rightarrow s_{41} \rightarrow s_{52}$	679,317	
	$s_{11} \rightarrow s_{21} \rightarrow s_{32} \rightarrow s_{42} \rightarrow s_{51}$	528,705	
	$s_{11} \rightarrow s_{21} \rightarrow s_{32} \rightarrow s_{42} \rightarrow s_{52}$	528,414	
…	...	…	…
DS_6	$s_{12} \rightarrow s_{29} \rightarrow s_{33} \rightarrow s_{42} \rightarrow s_{52}$	525,922	598,060
	$s_{12} \rightarrow s_{29} \rightarrow s_{33} \rightarrow s_{41} \rightarrow s_{52}$	669,375	
	$s_{12} \rightarrow s_{29} \rightarrow s_{33} \rightarrow s_{41} \rightarrow s_{51}$	669,766	

	s_{12}→s_{29}→s_{33}→s_{42}→s_{51}	526,153	
	s_{12}→s_{29}→s_{31}→s_{41}→s_{51}	670,268	
	s_{12}→s_{29}→s_{31}→s_{41}→s_{52}	669,977	
	s_{12}→s_{29}→s_{31}→s_{42}→s_{51}	526,655	
	s_{12}→s_{29}→s_{31}→s_{42}→s_{52}	526,364	
…	...	…	…

In the existing approaches for low-carbon product design, the comparison of alternative solutions is always at the same stage of product life cycle, which therefore cannot provide specific guidance on how to enumerate alternative solutions at each stage of product life cycle, and obtain the global optimum design across the entire product life cycle. In contrast, the proposed method not only quantitatively estimates the total carbon footprint in the life cycle, but also enumerates low-carbon design solutions. It considers the life cycle decisions and alternative life paths simultaneously. It provides both the ideal solution with the lowest carbon footprint and the practical solution with the lowest expected value of total carbon footprint incorporating uncertainties in the life cycle. The exact or relative reduction percentage of carbon footprint between the traditional and the proposed low-carbon design methods depends on the detailed carbon footprint data at the product life cycle.

6. Conclusions

In this paper, a low-carbon design approach based on the mapping between design solution and life cycle paths in product life cycle is proposed. A feature-based carbon footprint element model is proposed to quantitatively estimate the carbon footprint at each stage of product life cycle. A five-layer weighted directed graph-based carbon footprint model is developed to represent life cycle paths in the life cycle decision space. The proposed low-carbon design process is to provide an integrated approach that enumerates and combines alternative solutions at each stage of product life cycle with the lowest carbon footprint. The low-carbon design of a cold heading machine is used to demonstrate the proposed method.

Several extensions are needed in the future. First, the accuracy of the carbon footprint estimation is important for low-carbon design. The challenge is to quantify the uncertainty in the product life cycle, given various sources of uncertainties, such as the carbon footprint activity data, the carbon emission factors, etc. A robust analysis approach is needed to mitigate the impact of uncertainties. Data quality is one of the most important sources of uncertainty. Standards and rules of how data should be collected are needed in the carbon footprint evaluation. Second, how to choose an appropriate level of detail for features is another research issue, since the level of detail in the feature-based carbon footprint model is an important factor to calculate the carbon footprint. Designer should take a balanced approach to choose an appropriate level with the considerations of time required for data collection and analysis and the accuracy of carbon footprint. Third, further study of low-carbon conceptual design is needed, since the conceptual design has the most impact on the product's carbon footprint. Fourth, as the carbon footprint is just one of the several environmental impacts, and product design can have other objectives in addition to low-carbon, this approach could be applicable and extendable to other environmental impacts, which are evaluated by ISO14040.

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