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Algebraic interval constraint driven exploration in human-agent collaborative problem solving

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***Abstract*—To enable effective human-agent collaboration, new human-centric computing paradigms are needed. This paper presents a soft constraint representation scheme based on generalized intervals. With logically quantified intervals, semantics and intent can be integrated in numerical computing. The interpretable numerical results allow for better human-agent communication.**

I. Introduction

Human and computer possess different strengths in solving problems such as planning and searching. Compared to traditional autonomous agent-based planning and problem solving, new issues related to information representation and reasoning paradigms between human and computer need to be addressed in human-agent collaboration [1]. For instance, information should be represented towards human needs and capabilities. Human beings solve problems, including perception, abstraction, and understanding of real world, at different levels of granularity.

Second, asymmetric communication capabilities between human and computer exist. Human can capture computer's intent fairly quickly, while it is hard for computer to understand human intention. Dialog style communication between human and agent is needed during collaboration [2].

Third, collaboration within a human-agent team prefers soft computing. Reducing chances of conflict during the searching process of solution is important. In general, to enable effective agent-human collaboration, new human-centric soft computing paradigms are valuable.

As a subset of soft computing, granular computing [3, 4, 5] is an emerging conceptual and computing paradigm of information processing. It has been motivated by the urgent need for intelligent processing of empirical data into a humanly manageable abstract knowledge. Granular computing offers a landmark change from the current machine-centric to human-centric approach to information and knowledge, which involve interval and set theories (fuzzy set, rough set, etc.) in a highly comprehensive treatment. Philosophically, it is an intention of describing a way of thinking that relies on human ability to perceive the real world under various levels of granularity, which is captured and modeled with set-based logic and information representation schemes.

The essence of human-centric soft computation is computing with words as opposed to numbers in traditional machine-centric approach. Rather than applying traditional soft computing techniques such as fuzzy logic, neural computing, and machine learning, we use a generalized interval approach for human-agent collaborative problem solving. This is to allow soft constraint representation and solving in granular computing for human-agent interaction.

Interval, as a set of real values, is a good soft constraint representation mechanism in word-based human-human communication. For example, in giving direction, instead of "*turn 90.0 degree*", we usually say "*turn left*", which can be represented with interval as "*turn [70, 100] degree*". Instead of saying "*drive at a speed of 11.5 miles/hour*", we just say "*drive very slowly*", which may be captured as "*drive at a speed of [5,15] miles/hour*". In a human-centric computing environment, softness of constraints is one of the basic characteristics for human-agent communication.

In this paper, a new algebraic soft constraint representation scheme based on generalized intervals is proposed for human-agent collaboration. Extended from traditional set-based intervals, generalized intervals include logical quantifiers and provide interpretation of relations among intervals. This new scheme accommodates logic relations and semantics in mathematical forms. Intent capturing thus can be integrated into numerical computation.

The remainder of the paper is organized as follows. Section II briefly reviews interval analysis and generalized intervals. Section III describes the principles of interpretation in algebraic interval constraints. Section IV illustrates how soft constraints can be applied in human-agent collaborative path planning.

II. Background

A. Human-Agent Collaboration

In traditional human-robot interaction where humans remotely operate robots which have limited autonomy, human and robot have a master-slave relation. In human-agent collaboration, a cooperative and peer-to-peer relation exists between the two entities. Human and agent help each other to solve a common problem with the same goal. In previous work, Allen and Ferguson [6] developed an architecture of human-agent collaborative scheduling and planning. Breazeal et al. [7] demonstrated a framework of

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human-robot cooperative working and learning. Sidner et al. [8] studied the engagement process in collaborative conversation. Other research issues include collaborative control [9], motion planning for safety [10, 11], team centric autonomy [12, 13], and social order [14], etc.

However, in existing research of human-agent collaboration, soft constraint-driven approach for problem solving has not been considered. In human-centric communication, the property of softness in constraint imposition is important. In this paper, we propose a soft constraint driven problem solving approach based on generalized intervals.

B. Interval Analysis

Interval mathematics [15] is a generalization in which interval numbers replace real numbers, interval arithmetic replaces real arithmetic, and interval analysis replaces real analysis. The set of intervals is

$$\mathbb{IR} = \{[a,b] \mid a \in \mathbb{R}, b \in \mathbb{R}\} \tag{1}$$

Not only intervals solve the problem of representation for real numbers on a digital scale, but they are the most suitable way to represent uncertainties and errors in technical constructions, measuring, computations, and ranges of fluctuation and variation.

Interval analysis has been extensively used in reliable computing in computer science. In engineering fields, methods of interval analysis have been used in computer graphics [16], robust geometry construction and evaluation [17], robot control [18], imprecise structural analysis [19], finite-element formulation and analysis [20], soft constraint solving [21], and tolerance analysis and synthesis [22, 23].

Interval analysis captures intrinsic uncertainty and variance. However, it is based on a worst-case scenario as in traditional linear stack-up methods. The computational results usually are pessimistic in this variance addition scheme if dependency exists between variables. In contrast, modal interval analysis based on generalized intervals is an extension of the traditional interval analysis and differentiates different semantics of interval specification.

C. Modal Interval Analysis

Modal interval analysis (MIA) [24, 25, 26, 27, 28, 29, 30] is a logical and semantic extension of interval analysis. In MIA, a modal interval or generalized interval is not restricted to ordered bounds. Operations are defined in Kaucher arithmetic [31].

A modal interval or generalized interval $\mathbf{x} := [\underline{x}, \overline{x}] \in \mathbb{KR}$ is called *proper* when $\underline{x} \le \overline{x}$ and called *improper* when $\underline{x} \ge \overline{x}$. The set of proper intervals is denoted by $\mathbb{IR} = \{[\underline{x}, \overline{x}] \mid \underline{x} \le \overline{x}\}$, and the set of improper interval is $\overline{\mathbb{IR}} = \{[\underline{x}, \overline{x}] \mid \underline{x} \ge \overline{x}\}$. Given a generalized interval $\mathbf{x} = [\underline{x}, \overline{x}] \in \mathbb{KR}$, two operators *pro* and *imp* return proper and improper values respectively, defined as

$$\operatorname{pro}\mathbf{x} := [\min(\underline{x}, \overline{x}), \max(\underline{x}, \overline{x})] \tag{2}$$

$$\operatorname{imp}\mathbf{x} := [\max(\underline{x}, \overline{x}), \min(\underline{x}, \overline{x})] \tag{3}$$

The relationship between proper and improper intervals is established with the operator *dual*:

$$\operatorname{dual}\mathbf{x} := [\overline{x}, \underline{x}] \tag{4}$$

For example, $\mathbf{x} = [-1,1]$ and $\mathbf{y} = [1,-1]$ are both valid intervals. While $\mathbf{x}$ is a proper interval, $\mathbf{y}$ is an improper one. $\mathbf{x}$ and $\mathbf{y}$ have the relationship $\mathbf{x} = \operatorname{dual}\mathbf{y}$.

Given a set of closed intervals of real numbers in $\mathbb{R}$, and the set of logical existential ($\exists$) and universal ($\forall$) quantifiers, the semantics of a generalized interval $\mathbf{x} \in \mathbb{KR}$ is denoted by $(\mathrm{Q}_{\mathbf{x}} x \in \operatorname{pro}\mathbf{x})$ where $\mathrm{Q}_{\mathbf{x}} \in \{\exists, \forall\}$. An interval $\mathbf{x} \in \mathbb{KR}$ is called *existential* if $\mathrm{Q}_{\mathbf{x}} = \exists$. Otherwise, it is called *universal* if $\mathrm{Q}_{\mathbf{x}} = \forall$.

The uniqueness of generalized intervals is its logic extension from classical intervals. With this extension, MIA is able to model problems on a logical basis and to obtain the interval functional evaluations for mathematical models. The implications of constraint stacking can be modeled based on interpretability principles, as described in Section III.

III. Interpretability Principles

If a real relation $z = f(x_1, \cdots, x_n)$ is extended to the interval relation $\mathbf{z} = \mathbf{f}(\mathbf{x}_1, \cdots, \mathbf{x}_n)$, the interval relation $\mathbf{z}$ is interpretable if there is a semantic relation

$$(\mathrm{Q}_1 x_1 \in \operatorname{pro}\mathbf{x}_1)\ldots(\mathrm{Q}_n x_n \in \operatorname{pro}\mathbf{x}_n)(\mathrm{Q}_{\mathbf{z}} z \in \operatorname{pro}\mathbf{z}) \quad (5)$$
$$(z = f(x_1, \cdots, x_n))$$

As the basis of interpretation, two interval extensions of a real function $f(x): \mathbb{R}^n \to \mathbb{R}$, so-called semantic interval functions, are defined in min-max form as

$$f^*(\mathbf{x}) := \tag{6}$$
$$[\min_{x_p \in \operatorname{pro}\mathbf{x}_p} \max_{x_i \in \operatorname{pro}\mathbf{x}_i} f(x_p, x_i), \max_{x_p \in \operatorname{pro}\mathbf{x}_p} \min_{x_i \in \operatorname{pro}\mathbf{x}_i} f(x_p, x_i)]$$

$$f^{**}(\mathbf{x}) := \tag{7}$$
$$[\max_{x_i \in \operatorname{pro}\mathbf{x}_i} \min_{x_p \in \operatorname{pro}\mathbf{x}_p} f(x_p, x_i), \min_{x_i \in \operatorname{pro}\mathbf{x}_i} \max_{x_p \in \operatorname{pro}\mathbf{x}_p} f(x_p, x_i)]$$

where (x_p, x_i) is the component splitting corresponding to interval vector $\mathbf{x} = (\mathbf{x}_p, \mathbf{x}_i)$, with sub-vectors $\mathbf{x}_p$ and $\mathbf{x}_i$ containing proper and improper components respectively. Important properties of interpretability are available and proved based on these two semantic interval functions.

Theorem 3.1 [24] Given a continuous function $f(\mathbf{x}): \mathbb{R}^n \to \mathbb{R}$ and a generalized interval vector $\mathbf{x} \in \mathbb{KR}^n$, if there exists an interval $\mathbf{f}(\mathbf{x}) \in \mathbb{KR}$, then

$$f^*(\mathbf{x}) \subseteq \mathbf{f}(\mathbf{x}) \Leftrightarrow (\forall x_p \in \operatorname{pro}\mathbf{x}_p)(\mathrm{Q}_{\mathbf{f}} z \in \operatorname{pro}\mathbf{f}(\mathbf{x})) \quad (8)$$
$$(\exists x_i \in \operatorname{pro}\mathbf{x}_i)(z = f(x_p, x_i))$$

Theorem 3.2 [24] Given a continuous function $f(\mathbf{x}): \mathbb{R}^n \to \mathbb{R}$ and a generalized interval vector $\mathbf{x} \in \mathbb{KR}^n$, if there exists an interval $\mathbf{f}(\mathbf{x}) \in \mathbb{KR}$, then

$$f^{**}(\mathbf{x}) \supseteq \mathbf{f}(\mathbf{x}) \Leftrightarrow \left(\forall x_i \in \text{pro}\,\mathbf{x}_i\right)\left(\mathrm{Q}_{\text{dual}\,\mathbf{f}}\, z \in \text{pro}\,\mathbf{f}(\mathbf{x})\right) \left(\exists x_p \in \text{pro}\,\mathbf{x}_p\right)\left(z = f(x_p, x_i)\right) \quad (9)$$

Let $f(\mathbf{x}): \mathbb{R}^n \to \mathbb{R}$ be a rational continuous function. Its modal rational extension $\mathbf{f}: \mathbb{KR}^n \to \mathbb{KR}$ replaces the real variables of f with generalized interval variables and real operators with interval operators. The semantics of a modal interval relation or function is embodied in the relation's syntax. The syntax of a function $f(x_1, \ldots, x_n): \mathbb{R}^n \to \mathbb{R}$ can be represented by a syntax tree. For example, the syntax tree of $f_1 = \left|x_1 + x_2\right| / \left(x_1 - x_2\sqrt{x_3}\right)$ is shown in Fig. 1. A component x_i is uni-incident in the function $f(x_1, \ldots, x_n)$ if it occupies only one leaf of the syntax tree, such as x_3 in f_1. Otherwise, it is multi-incident, such as x_1 and x_2 in f_1. Leaves and branches of the syntax tree are connected with either one-variable operators such as $|\,|$ and $\sqrt{\ }$, or two-variable operators such as $+, -, \times, /$.

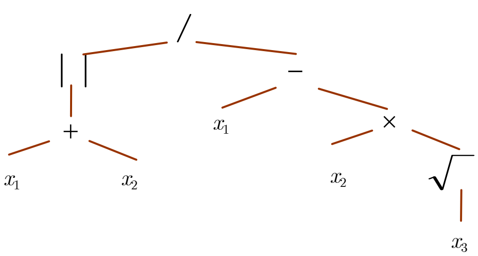

Fig. 1. The syntax tree of $f_1 = \left|x_1 + x_2\right| / \left(x_1 - x_2\sqrt{x_3}\right)$

A. *Uni-Incident Interpretation*

Theorem 3.3 [24] For a modal rational function $\mathbf{f}(\mathbf{x}): \mathbb{KR}^n \to \mathbb{KR}$, if all arguments of $\mathbf{f}(\mathbf{x})$ are uni-incident, then

$$f^{*}(\mathbf{x}) \subseteq \mathbf{f}(\mathbf{x}) \subseteq f^{**}(\mathbf{x}) \quad (10)$$

From Theorems 3.1, 3.2, and 3.3, we know modal rational functions of uni-incident variables are interpretable. For example, $f(x, y) = x + y$ is considered for $x \in [1,3]$ and $y \in [2,5]$.

$$\mathbf{f}([1,3],[2,5]) = [1,3] + [2,5] = [3,8],$$
$$\mathbf{f}([1,3],[5,2]) = [1,3] + [5,2] = [6,5],$$
$$\mathbf{f}([3,1],[2,5]) = [3,1] + [2,5] = [5,6],$$
$$\mathbf{f}([3,1],[5,2]) = [3,1] + [5,2] = [8,3],$$

have the meanings of

$$(\forall x \in [1,3])(\forall y \in [2,5])(\exists z \in [3,8])(z = x + y),$$
$$(\forall x \in [1,3])(\forall z \in [5,6])(\exists y \in [2,5])(z = x + y),$$
$$(\forall y \in [2,5])(\exists x \in [1,3])(\exists z \in [5,6])(z = x + y),$$
$$(\forall z \in [3,8])(\exists x \in [1,3])(\exists y \in [2,5])(z = x + y),$$

respectively.

B. *Multi-Incident Interpretation*

Theorem 3.4 [24] For a modal rational function $\mathbf{f}(\mathbf{x}): \mathbb{KR}^n \to \mathbb{KR}$, if there are multi-incident improper arguments in $\mathbf{f}(\mathbf{x})$ and $\mathbf{t}^{*}(\mathbf{x})$ is obtained from $\mathbf{x}$, by transforming, for every multi-incident improper component, all incidences but one into its dual, then $f^{*}(\mathbf{x}) \subseteq \mathbf{f}(\mathrm{t}^{*}(\mathbf{x}))$.

Theorem 3.5 [24] For a modal rational function $\mathbf{f}(\mathbf{x}): \mathbb{KR}^n \to \mathbb{KR}$, if there are multi-incident proper arguments in $\mathbf{f}(\mathbf{x})$ and $\mathbf{t}^{**}(\mathbf{x})$ is obtained from $\mathbf{x}$, by transforming, for every multi-incident proper component, all incidences but one into its dual, then $f^{**}(\mathbf{x}) \supseteq \mathbf{f}(\mathrm{t}^{**}(\mathbf{x}))$.

From Theorems 3.1, 3.2, 3.4, and 3.5, modal rational functions of multi-incident variables are interpretable with some modifications. For example, $f(x, y) = xy/(x + y)$ is extended to $\mathbf{x} = [-1,3]$ and $\mathbf{y} = [15,7]$.

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) = [-1,3] \times [15,7] / ([-1,3] + [15,7]) = [-0.5,1.5]$$

is not interpretable, whereas

$$\mathbf{f}(\mathbf{t}^{*}(\mathbf{x}, \mathbf{y})) = [-1,3] \times [15,7] / ([-1,3] + [7,15]) = [-1.1667,3.5],$$
$$\mathbf{f}(\mathbf{t}^{*}(\mathbf{x}, \mathbf{y})) = [-1,3] \times [7,15] / ([-1,3] + [15,7]) = [-1.0715,3.2143],$$
$$\mathbf{f}(\mathbf{t}^{**}(\mathbf{x}, \mathbf{y})) = [-1,3] \times [15,7] / ([3,-1] + [15,7]) = [-0.3889,1.1667],$$
$$\mathbf{f}(\mathbf{t}^{**}(\mathbf{x}, \mathbf{y})) = [3,-1] \times [15,7] / ([-1,3] + [15,7]) = [4.5,-1.5]$$

are interpretable. They are interpreted as

$$(\forall x \in [-1,3])(\exists y \in [7,15])(\exists z \in [-1.1667,3.5])(z = xy/(x + y)),$$
$$(\forall x \in [-1,3])(\exists y \in [7,15])(\exists z \in [-1.0715,3.2143])(z = xy/(x + y)),$$
$$(\forall y \in [7,15])(\forall z \in [-0.3889,1.1667])(\exists x \in [-1,3])(z = xy/(x + y)),$$
$$(\forall y \in [7,15])(\exists x \in [-1,3])(\exists z \in [-1.5,4.5])(z = xy/(x + y))$$

respectively.

Following the interpretability principles, we can formulate interval constraints in human-agent collaborative problem solving. In order to achieve specific semantics, different modalities can be assigned to interval variables accordingly. The constraint formulation process in human-agent communication is described in Section IV.

IV. Human-Agent Communication Based on Soft Constraints

Combination of natural languages and gestures is the natural communication method for human beings. Thus computing with words and perception requires extensive semantics and intent representation. Generalized intervals provide a good logic embedding mechanism for semantics embodiment for numerical computation. We use spatial relationships and reasoning to illustrate the concept of algebraic soft constraint representation, since spatial language is heavily used in task-level planning, searching, as well as navigation for robots. In these scenarios, collaborative environments with a combination of verbal commands and synthetic gesture aids such as personal digital assistant (PDA) are commonly used.

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A. *Intent Capturing and Semantics Modeling*

Intent capturing is one of the challenges in human-agent communication. There are two levels of intent: informative and communicative. Informative intent is the abstract intention in the plan, and it contains the meaning of plan. Communicative intent is manifested during the implementation, and it includes the meaning of planner. Task level human-agent communication focuses more on communicative intent.

In the algebraic interval constraint representation, communicative intent is embodied as specifications or constraints. For example, as illustrated in Fig. 2, when human asks robot to "*go that direction to find the door*", natural language is inherently imprecise. In our soft constraint representation scheme, the direction and distance are ranges and represented as intervals. Thus, the interval constraint capturing the relation between variables is

$$\mathbf{x}\cos\boldsymbol{\theta} = \mathbf{d}\,,$$

which is the interval extension of the real constraint. The direction and distance to move and the distance of the door are all given as interval ranges instead of rigid real numbers, which facilitate human-centric communication and computation with coarse-grained abstraction.

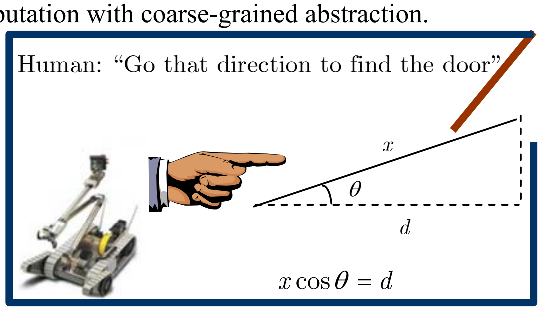


Fig. 2. Soft constraint is used in human-agent communication

The advantage of the algebraic interval constraint representation is that it allows us to capture some subtle semantics difference as an important component of communicative intent. In the example of Fig. 2, "*go that direction to find the door*" means the traveling distance x is to be determined and the semantics is

$$(\forall\theta \in \boldsymbol{\theta})(\forall d \in \mathbf{d})(\exists x \in \mathbf{x})(x\cos\theta = d) \tag{11}$$

However, "*go that direction, you may find a door*" means differently. The semantics is represented as

$$(\forall\theta \in \boldsymbol{\theta})(\forall x \in \mathbf{x})(\exists d \in \mathbf{d})(x\cos\theta = d) \tag{12}$$

Similarly, "*go a direction, you need to find the door*" can be captured as

$$(\forall d \in \mathbf{d})(\forall x \in \mathbf{x})(\exists\theta \in \boldsymbol{\theta})(x\cos\theta = d) \tag{13}$$

In this sense, intent and semantics are represented by interval-based predicates. If interval values with different modalities are given, different range estimation from computation will be derived. For instance, if we need to express the semantics of (11), the interval values are $\boldsymbol{\theta} = [0.2999, 0.4001] \in \mathbb{IR}$, $\mathbf{x} = [43.4281, 31.4026] \in \overline{\mathbb{IR}}$, and $\mathbf{d} = [40.000, 30.000] \in \overline{\mathbb{IR}}$. The numerical relation is $[43.4281, 31.4026] \times \cos([0.2999, 0.4001]) = [40.000, 30.000]$.

In this case, ranges of $\boldsymbol{\theta}$ and $\mathbf{d}$ are predetermined a priori by, e.g., sensors, whereas $\mathbf{x}$ is the range derived a posteriori by the constraint. However, if we want to express the semantics of (12), different interval values will be assigned or derived with different modalities. An example is $\boldsymbol{\theta} = [0.2999, 0.4001] \in \mathbb{IR}$, $\mathbf{x} = [32.5711, 41.8701] \in \mathbb{IR}$, and $\mathbf{d} = [30.000, 40.000] \in \mathbb{IR}$. The numerical relation is $[32.5711, 41.8701] \times \cos([0.2999, 0.4001]) = [30.000, 40.000]$. Here, the range of $\boldsymbol{\theta}$ may be predetermined a priori through sensing and $\mathbf{x}$ may be predetermined through, e.g., the estimation of battery life.

In general, within an algebraic interval constraint, an interval is called *a priori* estimation if it has a universal modality and called *a posteriori* estimation if it has an existential modality. Depending on application contexts, intervals are assigned as a priori estimations if they have the semantics of "predetermined", "crucial", "unchangeable", etc., in contrast to a posteriori estimations that have the semantics of "derived", "adjustable", "flexible", etc.

B. *Constraint Chain Formulation*

The semantic richness of generalized intervals provides a new computing mechanism with mixed symbolic and numerical computation. In a human-agent collaboration environment, information granule can be captured by interval values and associated logical quantifiers.

Soft constraints are formulated within specific collaboration scenarios. Problems need to be solved based on formulation of soft constraint chains, which are constructed as a result of conversational human-computer dialogs. As more constraints are added into the chain, solution space is narrowed down, and granularity of information increases.

Here, the constraint chain formulation process is illustrated with a collaborative path planning between human and robot in the example of Fig. 3. The objective is to find a path for the robot to navigate from a current position to a target. Suppose human suggests a path from any position $\mathbf{p}_{i-1}$ to a next position $\mathbf{p}_i$ with gestures, which are depicted as dash arrows in Fig. 3. Based on this approximated path, the robot needs to find a detailed and implementable one that will avoid collisions. The robot detects obstacle positions $\mathbf{r}_i$'s between two positions. Some intermediate positions $\mathbf{q}_i$'s are generated to avoid collision. Because of the vagueness of gestures, the positions $\mathbf{p}_i$'s are represented as interval boxes. With the consideration of errors and uncertainties involved in sensing, the obstacle positions $\mathbf{r}_i$'s that robot detects are also interval boxes. The derivation of intermediate positions $\mathbf{q}_i$'s is based on

$$\begin{cases} q_{ix} = r_{ix} + (-1)^v s(p_{iy} - p_{(i-1)y}) & (14) \\ q_{iy} = r_{iy} - (-1)^v s(p_{ix} - p_{(i-1)x}) & (15) \end{cases}$$

where s is the safety ratio to bypass obstacles; $v = 0$ if robot passes from the right side of an obstacle, and $v = 1$ if from the left side.

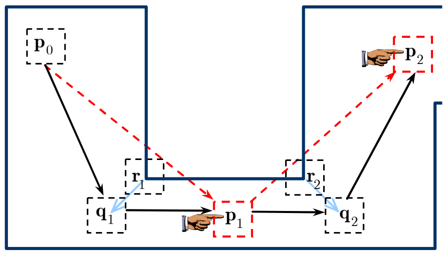


Fig. 3. An example of human-agent collaborative path planning

Based on the suggested path from human, robot computes intermediate interval positions and generates feasible paths. With the interpretability principles, logic relations among positions in the calculated path can be interpreted. The semantics of the relations is helpful for human and robot to understand the consequence of path planning.

The arithmetic of generalized intervals is implemented in Matlab and integrated with INTLAB [32], a toolbox for the classical interval analysis. The example of collaborative path planning in Fig. 3 is developed with a graphical user interface that uses mouse clicks to simulate synthetic gestures. The left-mouse and right-mouse clicks represent proper and improper intervals respectively. Users specify the positions $\mathbf{p}_i$'s. The derivation of intermediate positions $\mathbf{q}_i$'s is based on (14) and (15).

For instance, if considering $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{r}_1$, and $\mathbf{q}_1$ with the input as

$$\mathbf{s} = [0.1799, 0.2201] \in \mathbb{IR}$$
$$\mathbf{r}_1 = ([0.8499, 0.9501], [0.5599, 0.6401]) \in \mathbb{IR}^2$$
$$\mathbf{p}_0 = ([0.2969, 0.3370], [1.6814, 1.7215]) \in \mathbb{IR}^2 ,$$
$$\mathbf{p}_1 = ([1.4499, 1.8501], [0.0999, 0.5000]) \in \mathbb{IR}^2$$

and modal interval extension

$$\begin{cases} \mathbf{q}_{1x} = \mathbf{r}_{1x} + \mathbf{s}(\mathbf{p}_{1y} - \mathbf{p}_{0y}) \\ \mathbf{q}_{1y} = \mathbf{r}_{1y} - \mathbf{s}(\mathbf{p}_{1x} - \mathbf{p}_{0x}) \end{cases}$$

we have the output $\mathbf{q}_1 = ([0.4932, 0.7374], [0.2183, 0.4397])$. The interpretations are

$$(\forall p_{0y} \in [1.6814, 1.7215])(\forall p_{1y} \in [0.0999, 0.5000])$$
$$(\forall s \in [0.1799, 0.2201])(\forall r_{1x} \in [0.8499, 0.9501])$$
$$(\exists q_{1x} \in [0.4932, 0.7374])(\text{Equ.(14) holds})$$

and

$$(\forall p_{0x} \in [0.2969, 0.3370])(\forall p_{1x} \in [1.4499, 1.8501])$$
$$(\forall s \in [0.1799, 0.2201])(\forall r_{1y} \in [0.5599, 0.6401])$$
$$(\exists q_{1y} \in [0.2183, 0.4397])(\text{Equ.(15) holds})$$

Combining these two, the result has the meaning of

$$(\forall p_0 \in \mathbf{p}_0)(\forall p_1 \in \mathbf{p}_1)(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)(\exists q_1 \in \mathbf{q}_1)$$
$$(\text{A path is formed})$$

Here, $\mathbf{q}_1$ is a "flexible" or "soft" position where the robot can adjust itself for the next step. In contrast, $\mathbf{p}_0$ and $\mathbf{p}_1$ are "rigid" positions where the positions are critical and cannot be adjusted. The numerical result is shown in Fig. 4-a.

Suppose the position of $\mathbf{p}_1$ becomes flexible, that is,

$$\mathbf{p'}_1 = ([1.8501, 1.4499], [0.5000, 0.0999]) \in \overline{\mathbb{IR}}^2$$

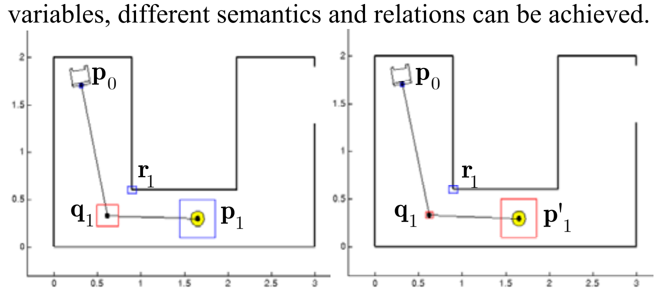
the variation range of $\mathbf{q}_1$ can be reduced to

$$\mathbf{q'}_1 = ([0.5812, 0.6654], [0.3063, 0.3677])$$

The result is shown in Fig. 4-b and interpreted as

$$(\forall p_0 \in \mathbf{p}_0)(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)(\exists p_1 \in \text{pro}\,\mathbf{p'}_1)(\exists q_1 \in \mathbf{q'}_1)$$
$$(\text{A path is formed})$$

Assigning different combinations of modalities to interval variables, different semantics and relations can be achieved.



(a)

$$(\forall p_0 \in \mathbf{p}_0)(\forall p_1 \in \mathbf{p}_1)(\forall s \in \mathbf{s})$$
$$(\forall r_1 \in \mathbf{r}_1)(\exists q_1 \in \mathbf{q}_1)$$
$$(\text{A path is formed})$$

(b)

$$(\forall p_0 \in \mathbf{p}_0)(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)$$
$$(\exists p_1 \in \text{pro}\,\mathbf{p'}_1)(\exists q_1 \in \mathbf{q'}_1)$$
$$(\text{A path is formed})$$

Fig. 4. Numerical results with semantics, where blue and red boxes represent a priori and a posteriori estimations respectively

If multiple paths are involved, the segments can be computed separately. Then the concatenation of these segments forms a chain of paths with a combination of semantics. For example, the concatenation of

$$(\forall p_0 \in \mathbf{p}_0)(\forall p_1 \in \mathbf{p}_1)(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)(\exists q_1 \in \mathbf{q}_1)$$
$$(\text{Path 1 is formed})$$

and

$$(\forall p_1 \in \mathbf{p}_1)(\forall p_2 \in \mathbf{p}_2)(\forall s \in \mathbf{s})(\forall r_2 \in \mathbf{r}_2)(\exists q_2 \in \mathbf{q}_2)$$
$$(\text{Path 2 is formed})$$

generates

$$(\forall p_0 \in \mathbf{p}_0)(\forall p_1 \in \mathbf{p}_1)(\forall p_2 \in \mathbf{p}_2)(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)$$
$$(\forall r_2 \in \mathbf{r}_2)(\exists q_1 \in \mathbf{q}_1)(\exists q_2 \in \mathbf{q}_2)(\text{Paths are formed})$$

as shown in Fig. 5-a. The concatenation of

$$(\forall p_0 \in \mathbf{p}_0)(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)(\exists p_1 \in \mathbf{p'}_1)(\exists q_1 \in \mathbf{q'}_1)$$
$$(\text{Path 1 is formed})$$

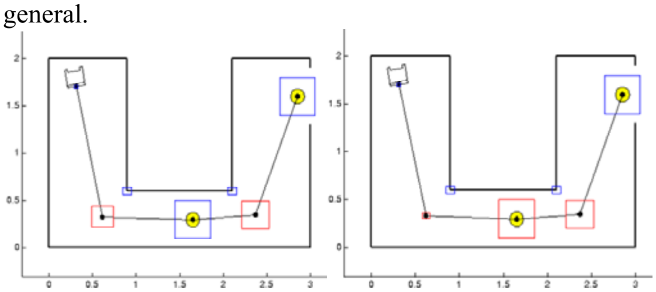
and

$$(\forall p_1 \in \mathbf{p'}_1)(\forall p_2 \in \mathbf{p}_2)(\forall s \in \mathbf{s})(\forall r_2 \in \mathbf{r}_2)(\exists q_2 \in \mathbf{q'}_2)$$
$$(\text{Path 2 is formed})$$

generates

$$(\forall p_0 \in \mathbf{p}_0)(\forall p_2 \in \mathbf{p}_2)(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)(\forall r_2 \in \mathbf{r}_2)$$
$$(\exists p_1 \in \mathbf{p'}_1)(\exists q_1 \in \mathbf{q'}_1)(\exists q_2 \in \mathbf{q'}_2)(\text{Paths are formed})$$

as shown in Fig. 5-b. Notice that the concatenation of $\forall p \in \mathbf{p}$ and $\forall p \in \mathbf{p}$ is $\forall p \in \mathbf{p}$, and the concatenation of $\forall p \in \mathbf{p}$ and $\exists p \in \mathbf{p}$ is $\exists p \in \mathbf{p}$. However, the concatenation of $\exists p \in \mathbf{p}$ and $\exists p \in \mathbf{p}$ is not $\exists p \in \mathbf{p}$ in general.



(a)

$(\forall p_0 \in \mathbf{p}_0)(\forall p_1 \in \mathbf{p}_1)(\forall p_2 \in \mathbf{p}_2)$
$(\forall s \in \mathbf{s})(\forall r_1 \in \mathbf{r}_1)(\forall r_2 \in \mathbf{r}_2)$
$(\exists q_1 \in \mathbf{q}_1)(\exists q_2 \in \mathbf{q}_2)$
(Paths are formed)

(b)

$(\forall p_0 \in \mathbf{p}_0)(\forall p_2 \in \mathbf{p}_2)(\forall s \in \mathbf{s})$
$(\forall r_1 \in \mathbf{r}_1)(\forall r_2 \in \mathbf{r}_2)(\exists p_1 \in \mathbf{p}'_1)$
$(\exists q_1 \in \mathbf{q}'_1)(\exists q_2 \in \mathbf{q}'_2)$
(Paths are formed)

Fig. 5. Concatenation of constraints forms a chain of paths

V. Concluding Remarks

In human-agent collaborative problem solving, human beings need to interactively give guidance to autonomous agents during searching, while agents perform computation concurrently. This interactivity can speed up the solution searching process. In this paper, we present a new soft constraint formulation mechanism based on generalized intervals, in which semantics and communicative intent are captured and embedded in numerical computation. This human-centric constraint-driven problem solving approach is illustrated with collaborative path planning. Interval values represent imprecise position information inherent in human dialogs. With existential and universal modalities, interpretable logic relations among intervals are integrated with numerical constraints. This creates a new possibility of improving human-agent communication with mixed symbolic and numerical computing.

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