100.578 Uncertainty Quantification & Analysis in Systems Modeling and Simulation

Exercise 3

Q1. A sensor that has stochastic behaviors has three possible states: *Busy* (state 1), *Idle* (state 2), and *Down* (state 3). The transition matrix is

$$\begin{bmatrix} 0.7 & 0.3 & \\ 0.3 & 0.4 & 0.3 \\ & 0.3 & 0.7 \end{bmatrix}$$

(1) Given an initial state vector $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, compute 20 iterations of state transitions (e.g. using Matlab) and show the respective state vectors at the 1st , 5th, 10th, 15th , and 20th iterations.

(2) Analytically find the stationary distribution of states without directly computing each step of transition.

Q2. Show that a finite-state discrete-time Markov chain with transition matrix Γ will converge to a stationary distribution of states is that each of Γ's eigenvalues, λ_i's, must be either $\lambda_i = 1$ or $|\lambda_i| < 1$.